BASIS AND APPLICABILITY OF β LIMITS: TO COMPACT STELLARATORS

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GENERAL ATOMICS

Relevance of MHD β Limits in Stellarators is Not Well Understood

- **MHD stability limits in Tokamaks is considered well understood:**
  - Ideal MHD predicts stability limits, growth rates, and mode structures in many situations
  - Fast, global instabilities are identified with disruptions and β collapse
  - Localized and weakly growing instabilities are identified with benign MHD activity: Edge Localized Modes (ELMs), Sawteeth, etc.
- **Stellarators however appear to violate MHD stability limits:**
  - Recent LHD and W7AS experiments exceeded predicted β limits
  - β appears to be limited by a soft limit of degrading confinement:

    β limits in the tokamak sense have not yet been observed
  - Some correlation is still observed between mode onset and linear stability threshold ⇒ Ideal MHD predictions do mean something

But Stellarators and Tokamaks have the same underlying physics based on Maxwell's Equations and Newtonian mechanics!

Tokamaks Provide Context for Understanding Role of MHD Stability in Stellarators

- **Tokamak studies show importance of distinguishing different β limits:**
  - Local ideal ballooning and interchange modes
  - Global ideal internal modes
  - Global ideal external modes
  - Resistive interchange modes
  - Equilibrium limits to β
- **And suggest how to proceed:**
  - Stability limits depend sensitively on the equilibrium details:

  $\Rightarrow$ **Equilibrium characterization is crucial to identifying the problem precisely**
  - Need to develop intuition for mode relevance in each case
    - # From experiments
    - # From nonlinear stability calculations
- **What criteria can we use right now?**

Tokamak Experience is Not So Different

- **Tokamaks also routinely violate some MHD stability limits:**
  - Limits are open to interpretation and are not always hard limits:
    - # Tokamaks routinely operate with $q < 1$, unstable to internal kink instability: $\Rightarrow$ Sawteeth
    - # Tokamak ballooning modes are not always devastating: $\Rightarrow$ Soft β limit
    - # In H-mode Tokamaks also routinely reach intermediate n stability limits: $\Rightarrow$ Generally benign ELMs
- **Tokamak stability limits depend sensitively on the equilibrium:**
  - Not sufficient to fit equilibrium to global discharge parameters

    $\Rightarrow$ **Stability can depend quite sensitively on profile details**
  - In Stellarators the ι profile is not normally measured at finite β
  - **Once it was measured, the q profile in Tokamaks was not what everyone thought it should be!**

Interpretation of Local Ideal Ballooning and Interchange Mode Limits

General consensus is that large Stellarator experiments routinely exceed local ideal ballooning and Mercier β limits

- **LHD Achieved β > 4% (Sakakibara EPS 2004):**
  - Heliotron configuration has a magnetic hill in the peripheral region
  - $\Rightarrow$ Violation of stability of ideal and resistive interchange modes is a concern but modes are not seen
- **W7-AS: Achieved β > 3.4% (Zarnstorff IAEA 2004):**
  - MHD activity in early medium β phase
  - Predicted ideal MHD local stability limit β ~ 2%

$\Rightarrow$ **Should these limits be ignored in design studies?**

Ignoring Local Limits is Consistent with Longstanding Tokamak Experience

- **Large Tokamak experiments also routinely operate with axis q below 1.0 violating Mercier stability**

  $\Rightarrow$ This is identical to the situation in Stellarators

- **Ballooning instability in Tokamaks appears to cause confinement saturation so that profiles do not exceed the local ballooning limit**

  May be some differences in Stellarators but absence of accurate equilibrium reconstruction precludes a definite conclusion

- **Open question for Stellarators: how do local ballooning mode solutions relate to global modes:**
  - Construction of global modes from local mode solutions may yield higher, more relevant β limits if local criteria are ignored (Ware EPS 04)
  - It seems appropriate to construct such solutions and use those to determine the ballooning limit

Interpretation of Global Ideal Internal Mode β Limits

- **Stellarators are beginning to distinguish physically relevant instabilities**
  - W7-X studies considering "physically relevant" modes as those with growth rate above a finite cutoff
  - LHD correlating observed modes with sufficiently large predicted radial width and growth rates above a finite cutoff
- **This View is Also Consistent with Tokamak Experience**
  - **Tokamaks routinely operate with several weakly unstable ideal global internal modes**
  - Ideal internal m/n = 1/1 mode generally weakly unstable if $q < 1$ ($\iota > 1$)
    - # 1/1 stability is dependent on a range of non-ideal contributions
    - # The 1/1 ideal instability is routinely ignored in stability calculations
  - Weakly growing "infernal" modes localized in low shear regions are sometimes observed but typically saturate and then decay

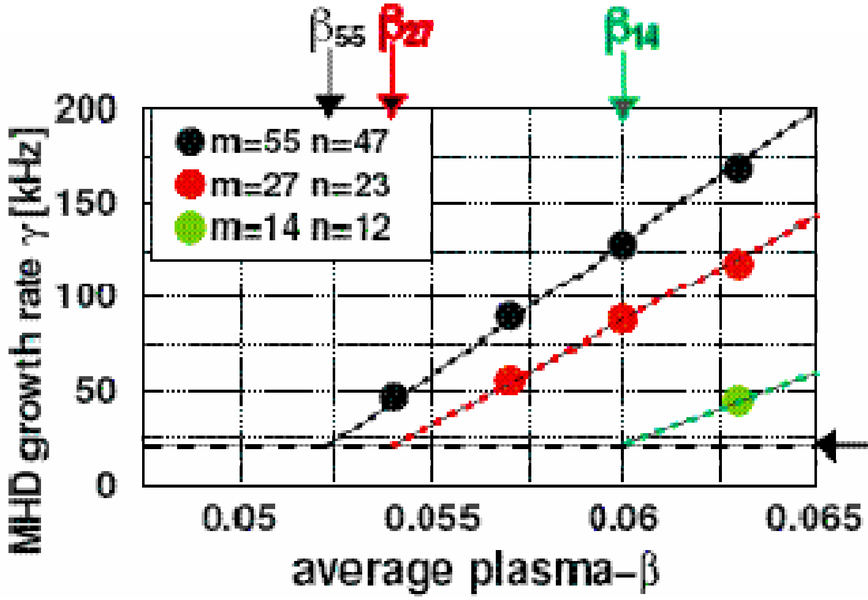
Stability to Physically Relevant Growth Rates Yields β Limit above 5% in W7-X

- **W7-AS experiments saw high β quiescent phase after an earlier startup with noticeable MHD activity:**
  - Study for W7-X compared CAS3D stability with W7-AS observations
- **Physically Relevant Growth Rates Considered to be > 20 khz (20 μs)**

  ⇒ **Calculated β limit is 5.25% (limited by high m modes)**

  ⇒ **For the low n modes only (m=14): calculated β limit ~ 6%**

Physical growth rates versus β

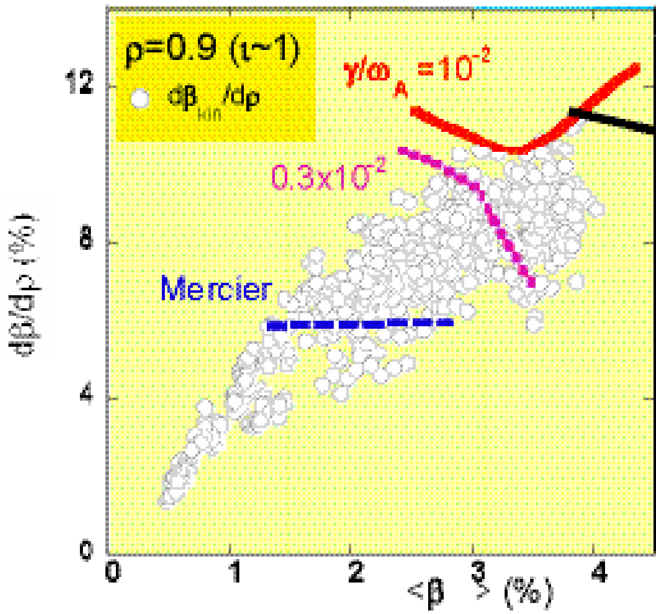


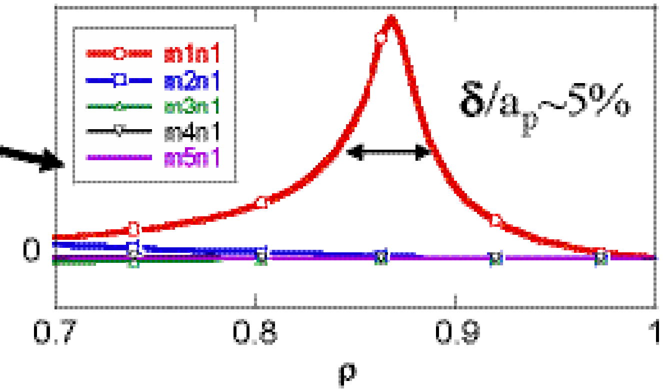
(C. Nuhrenberg: 30th EPS Conference on Contr. Fusion and Plasma Phys., St. Petersburg, 2003 ECA Vol. 27A, P-1.16)

Low m/n = 1/1, 2/3, and 2/5 Internal Modes Appear to Determine β Limit in LHD

- **In LHD several MHD modes (m/n = 1/1, 2/3, 2/5) are excited in edge region and spontaneously stabilized in turn as β increases:**
  - Profile flattening observed and contributes to MHD mode stabilization
  - These modes limit the pressure gradient in the peripheral region
- **Theoretical prediction suggests m/n = 1/1 mode has resonance around ρ = 0.9 and that this mode determines the β limit in LHD (Sakakibara EPS 2004)**
- **Actual β limit appears to correlate with a ‘big enough’ mode:**
  - **‘Big enough’ is defined by the radial mode width**

β Limit in LHD Appears To Correlate with Predicted Mode Width ~ 5% Minor Radius





Radial structure of ideal MHD mode calculated by **TERPSICHORE code**

Though beta value and its gradients reach the global mode unstable region where the predicted radial width of the ideal MHD mode reaches δ/a_p =5%,the disruptive degradation of global confinement has not been observed.

Interpretation of Global Ideal External Mode β limits

- **Several external mode types need to be distinguished:**
  - Global β driven modes
  - Current driven peeling modes
  - Current driven external kinks
  - ELMs
- **Global β driven modes expected to result in a true β limit**
- **Low n external kink and intermediate n peeling mode stability generally depends sensitively on edge conditions:**
  - Tokamak experience shows a good equilibrium characterization is needed to fully compare experiment and theory predictions:
    - # External "peeling mode" stability depends sensitively on vicinity of mode rational surface in vacuum
    - # Similar sensitivity to edge rational ι is observed in LHD

ELMs Are of Particular Significance Since They Are Observed in Stellarators And Tokamaks

ELMs do not directly result in β limits in either case!

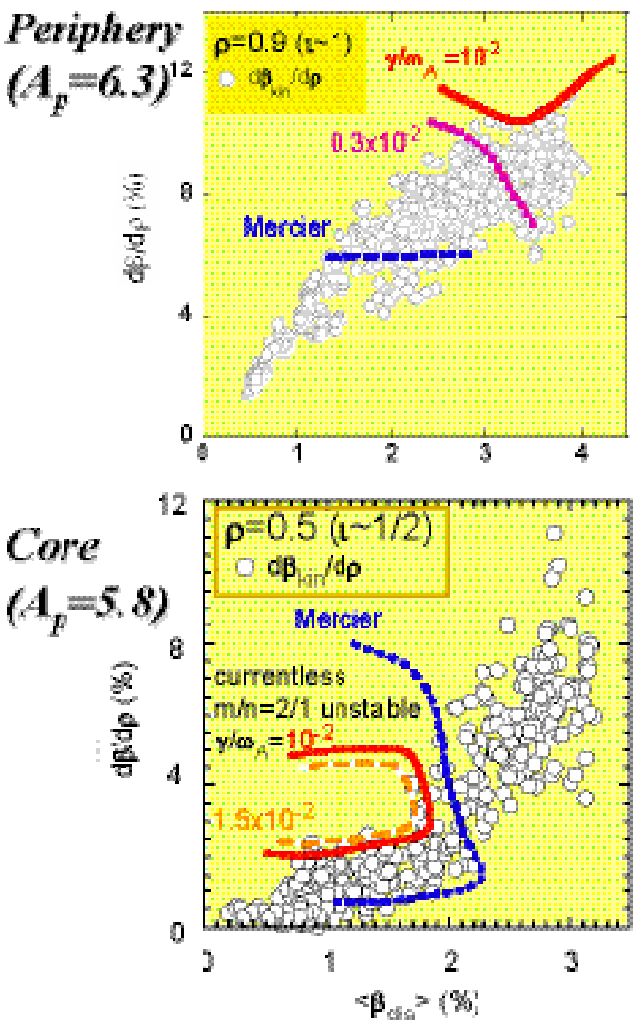
- **In Tokamaks ELMs appear to be primarily intermediate n ideal edge instabilities related to peeling modes:**
  - Mode is driven by combination of bootstrap current and pressure gradient from steep edge pressure gradient
  - Generally referred to as “peeling-balloning” modes
- **In Stellarators it is not clear ELMs are related to the same ideal edge modes:**
  - ELMs may be induced by resistive/ideal interchange modes
- **Note: Intuition from simple models can be misleading:**
  - Common intuition ⇒“peeling modes” are current driven modes related to finite edge q near a rational value
  - In divertor case $q \rightarrow \infty$ but “peeling modes” coupled to pressure driven modes still occur

  ⇒ Classic current driven ”peeling modes” do not exist in diverted equilibria but a coupled pressure/current driven version does exist - the so-called “peeling-ballooning mode”

Global Edge Stability Depends Strongly on Edge Conditions and Rational Edge Values

- **Plasma Boundary Has a Significant Influence on MHD Stability in Heliotrons (N. Nakajima JIFT 2005):**
  - Finite pressure gradient observed beyond LCFS
  - Inward shifted configurations have narrowest stochastic layer
  - Assuming average flux surfaces in stochastic region, configuration is predicted unstable for fixed boundary at β = 3%, but marginally stable for free boundary
  - At high β, growth rates decrease with increasing β due to boundary modification
- **Plasma behavior is affected by the rational surface existing at the plasma boundary in H Mode in LHD (S.Morita EPS 2004):**
  - Plasma edge behavior strongly affected by nearby ι = 1 surface
- **Proposed measure for the operational β limit in LHD from linear ideal MHD theory:**
  - **Maximum β occurs in a limited number of experiments where, for low n modes, $\gamma/\gamma_A \sim 10^{-2}$**

LHD Maximum β Reaches Value Where Predicted Low n Growth Rate Exceeds Critical Threshold



Though the observed pressure gradients are in non-linear saturation phases, a linear MHD theory could be a reference for more complicated non-linear analyses, and/or a criterion for a reactor design.

Peripheral region: the maxima of the achieved pressure gradients are less than $\gamma_{low\text{-}n}/\omega_A = 10^{-2}$.
Core region; the maxima of the achieved pressure gradients saturate against the contour of $\gamma_{low\text{-}n}/\omega_A = 1.5\times10^{-2}$ in the range of $\langle\beta_{dia}\rangle$ = 1~1.8%.

Roughly speaking, $\gamma_{low\text{-}n}/\omega_A$= 1~1.5x10^{-2} is considered a good index to determine the condition that the global ideal MHD instability limits the LHD operational regime.

For further verification, we need to extend the above comparative analyses between the experimental results and the theoretical prediction based on a linear theory to many magnetic configurations in LHD!!

(Watanabe IAEA 2004)

Interpretation of Resistive Mode β Limits

- **Observed edge MHD mode in LHD is thought to be resistive interchange mode (K. Toi EPS 2003):**
  - Dominant mode at L-H transition of LHD plasmas is m=2/n=3
  - Edge in magnetic hill (destabilizes resistive interchange) but high magnetic shear region (stabilizes ideal interchange)
- **Resistive interchange in LHD appears to be much like their ideal counterpart:**
  - Typically predicted to be unstable at low β
  - Does not seem to be limit β

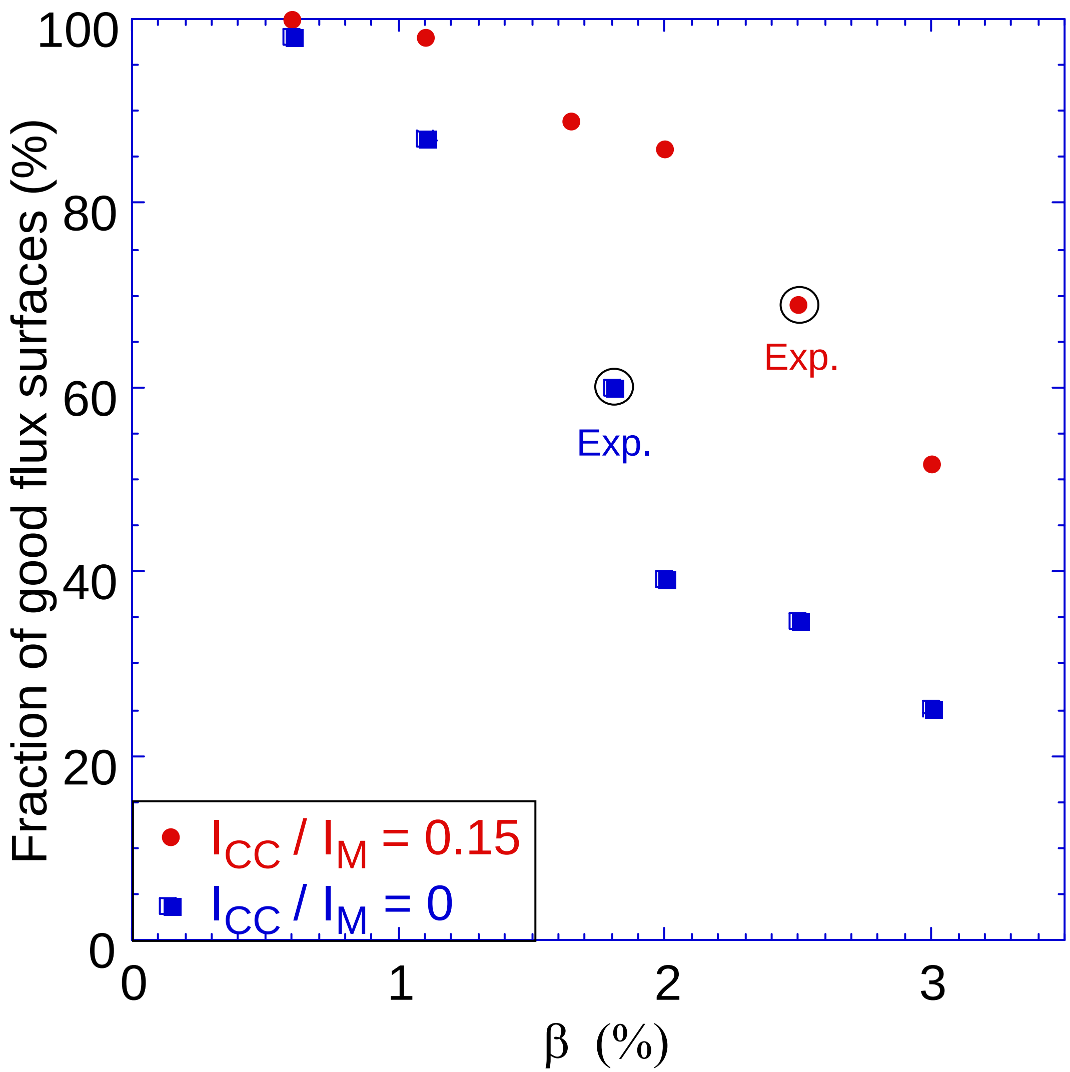
Interpretation of Equilibrium β Limits

- **Equilibrium β Limits are still a prime candidate for setting the operational β limit in Stellarators:**
  - Both LHD and W7-AS observe equilibrium degradation at high β
  - Maximum β in W7-AS appears to be limited by changes in confinement and not MHD activity
- **This is not necessarily in conflict with observations of MHD modes in Stellarator experiments at high β:**
  - The observed MHD may be a manifestation of the equilibrium degradation through island formation or:
  - The equilibrium degradation (island formation) may be a manifestation of the approach to an unstable situation

Equilibrium Degradation May Set W7-AS β Limit

- PIES equilibrium calculations indicate fraction of good surfaces drops with β
- Drop occurs at higher β for higher I_{CC} / I_M

Experimental β value correlates with loss of ~35% of minor radius to stochastic fields or islands



Summary: LHD and W7-AS MHD β Limit Status

- **In LHD and W7-AS β values achieved significantly exceed the Mercier interchange limit:**
  - Maximum volume-averaged β above 3.5% achieved in both
- **In LHD β appears to be limited by an m/n = 1/1 ideal limit (Watanabe IAEA 04)**
- **In W7-AS β appears to be limited by approach to the equilibrium limit (Zarnstorff IAEA 04)**
- **In either case ideal stability plays a direct or indirect role:**
  - **Degradation of the equilibrium is strongly associated with approach to MHD stability limits**
  - **Strongly growing ideal modes appear to provide a direct limit**

Stability Limits Can Depend Sensitively on the Equilibrium

- **It is not normally sufficient to fit the equilibrium to just the global characteristics of Tokamak discharges:**
  - One can obtain widely varying results depending on the form assumed for the current density and pressure profiles for similar global parameters
  - Profiles need to be measured accurately and used in reconstructing the equilibrium for the stability calculations
- **In Stellarators the equilibrium is believed to be known. But:**
  - The ι profile is often taken from the vacuum profile:

    $\Rightarrow$ **It may be different at finite β**
  - The pressure profile is not known as a function of flux:

    At most it is measured as a function of space and the mapping to flux space needed for the equilibrium depends on the ι profile
  - Given the sensitivity to the equilibrium, nested flux surfaces might be a poor approximation for stability even for small islands

Characterization of Experimental Stellarator Equilibria is Improving Rapidly

- **Realization that accurate equilibrium reconstructions are needed in Stellarators is now becoming more widespread:**
  - "In helical systems, the characteristics of MHD equilibrium, stability and transport with high β and large toroidal current are quite different from those in vacuum" (T. Yamaguchi EPS 2004)
  - "The careful reconstruction of the equilibrium with applying asymmetrical profile is required for understanding of the mechanism of this mode stabilization [from profile flattening at high β]" (S. Sakakibara EPS 2004)
- **New diagnostics are being developed and implemented at both LHD and W7-X for reconstructing pressure and current (ι) profiles:**

  ⇒ **In future one can determine more precisely which modes actually exceed predicted limits !**

One can then interpret the role of individual instabilities in determining operational β limit in Stellarators !

Physical Relevance Can be Studied by Considering Nonlinear Stability in Comparison With Experiments

- **Existence of a nested flux surface equilibrium can be considered as either an equilibrium or a stability problem**
  - Unstable equilibria with nested surfaces will evolve to a nearby non-nested surface state lower energy if physically possible

  $\Rightarrow$ **PIES, HINST, NSTAB,…may be useful as nonlinear stability tools!**

- **NSTAB nonlinear stability code exploits relation between equilibrium and stability by searching for bifurcated equilibria:**
  - Existence of discontinuities $\Rightarrow$ current sheet within nested flux surface approximation
  - Current sheets resolved in reality by formation of islands
  - Equilibria should be stable to profile preserving instabilities
  - Nonlinear stability evaluated by employing a mountain pass theorem with the search for bifurcated equilibria

Criteria appear to predict LHD and W7-AS β limits reasonably well

Some Important Distinctions Exist Between Tokamaks and Stellarators

- **Distinctions may produce superficially different behavior even if fundamentally MHD is valid in Tokamaks and Stellarators:**
  - Current and pressure profiles may be quite different between Tokamaks and Stellarators
  - Linear stability calculations generally assume nested flux surfaces

    ⇒ **In tokamaks this is normally an accurate assumption**
  - In Stellarators nested surfaces may not exist !

    ⇒ **Even non-nested surfaces might not exist: field may be stochastic !**
  - Relative roles of current and pressure in driving MHD instability may mean different observed behavior
- **Resolution requires testing predictions using discharge equilibria:**

  ⇒ **Detailed measurements of stellarator ι profiles are needed**

Compact Stellarators may be more Tokamak-like than conventional Stellarators !

Finite average current may or may not reproduce more closely Tokamak-like MHD behavior

Conclusion: Linear Stability Predictions With Nested Surfaces Can Be Used as Guide if Interpreted Properly

- **Distinction needs to be made between different mode types**
- **Local stability criteria should probably be ignored:**
  - There is little reason that infinite n should provide a physical limit
  - Finite n corrections appear to be large given the difference between the global code limits and the infinite n localized limits
- **Global MHD stability must be tested using reconstructed equilibria:**
  - Need to use the measured equilibrium profiles
  - May need to construct a non-nested flux surface equilibrium:
    - $\Rightarrow$ States with different prescriptions for the multiple values for p and j in different simply connected regions (islands etc.) are possible and may be physically accessible
  - Flux surfaces might not even exist
  - Actual profiles will be determined by transport and topology
- **MHD stability predictions need to be interpreted after testing using reconstructed equilibria against actual experiments**

How Should We Proceed? What Questions Remain?

- **To proceed for ARIES-CS design:**
  - **Ignore local stability criteria**
  - **Check linear global stability (TERPSICHORE) as guide to approximate limit**
  - **Monitor linear stability predictions against nonlinear predictions (NSTAB)**
  - **Check flux surface quality (PIES)**
- **Are nested surfaces a valid approximation for stability calculations:**
  - Does linear instability of a nested flux surface equilibrium simply result in benign nonlinear evolution to a ‘nearby’ non-nested state?
  - If nested surfaces are not valid, can the stability problem be formulated in terms of finding nonlinearly stable equilibria?
- **Nonlinear consequences crucial for interpreting stability calculations:**
  - Generally internal modes surrounded by a fairly robust and stable outer shell might be expected to be benign
  - Is there a way to quantify this without the full nonlinear calculation?

Further progress requires criteria to decide when linear instability of nested flux surface equilibria result in benign nonlinear evolution to ‘nearby’ states:

Requires direct comparison with experiments and nonlinear stability calculations