ASSESSMENT OF □ LIMITS FOR COMPACT STELLARATORS

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and

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All outrageous and just plain wrong statements are entirely those of the Primary Author!

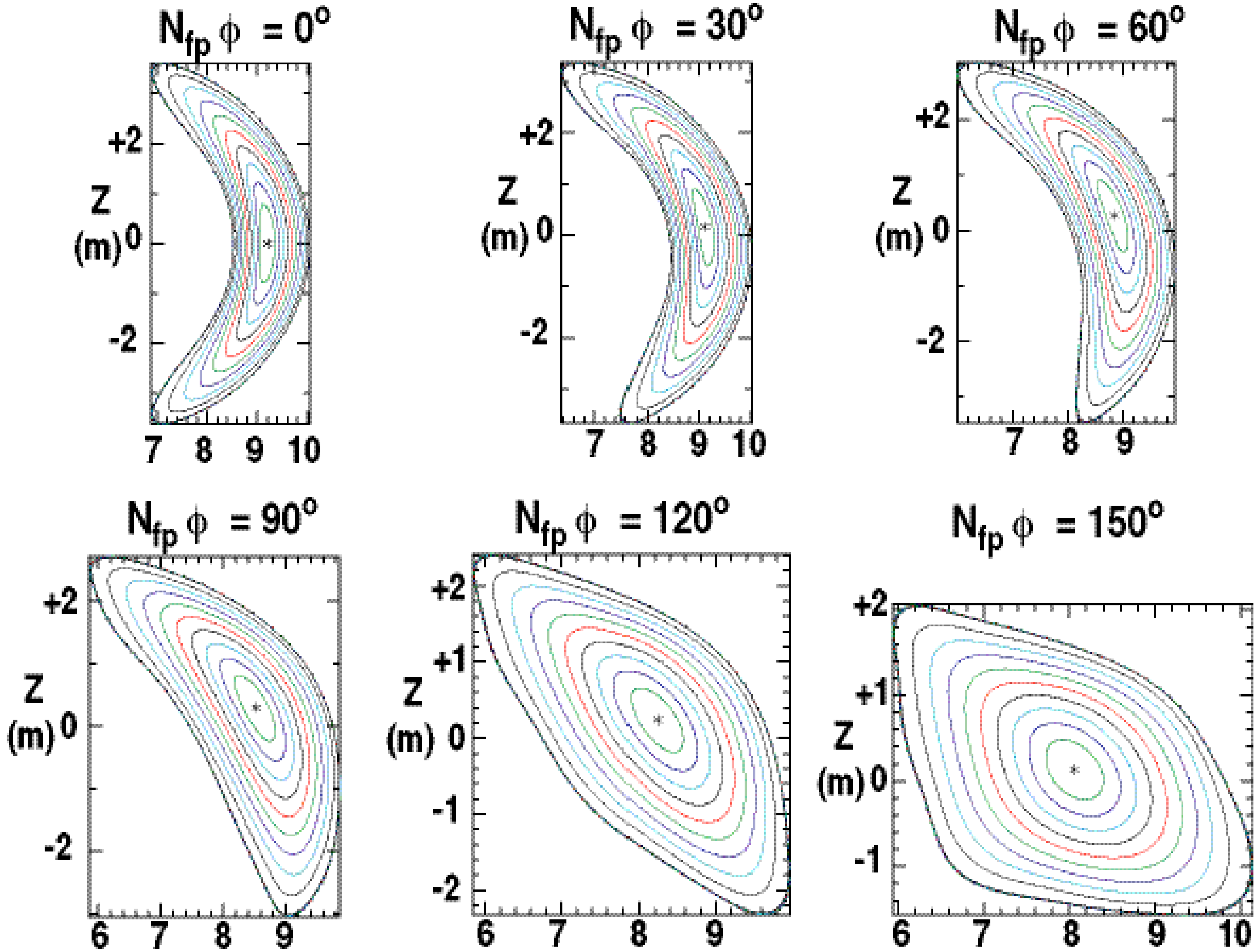
COMPACT STELLARATOR EQUILIBRIUM AND STABILITY ANALYSIS

- **Scaled-up NCSX equilibrium (Long-Poe Ku PPPL) reproduced using the GA version of the VMEC code:**
  - **Equilibrium $\langle \beta \rangle = 4.1\%$ and $A = 4.47$**
  - **Equilibrium toroidal flux contours for a series of toroidal angles covering a single field period**

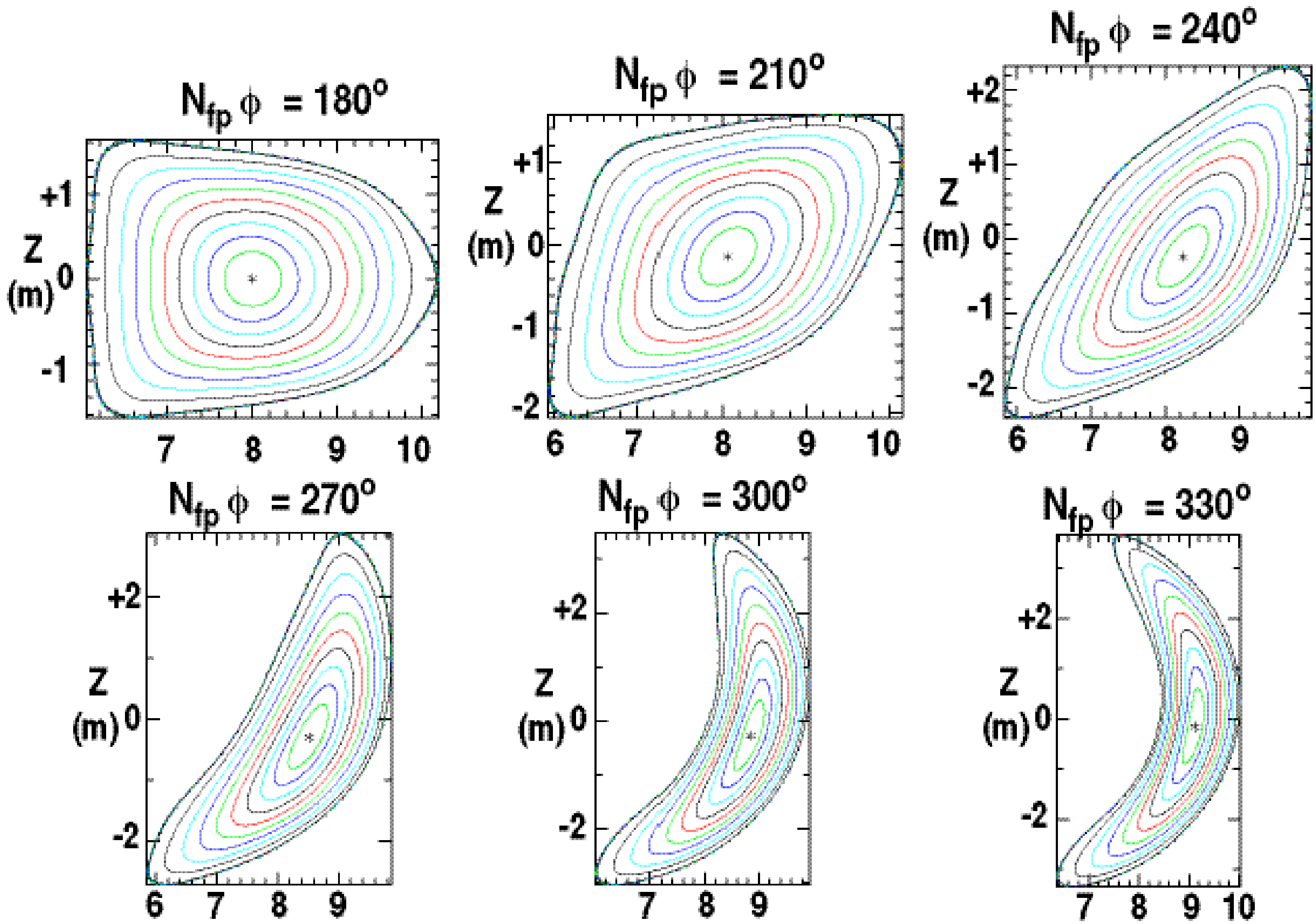
- **Stability results obtained for the three-period scaled up NCSX equilibrium**
  - **For this three field period stellarator mode coupling is confined to a single mode family represented by n = 1**
    - **No other toroidal mode families exist**

- **Stability results for the scaled-up equilibrium restricted to a range of moderately placed external conformal conducting walls**
  - **Between 1.7 and 2.7 times the minor radius of the plasma**
  - **For the wall in this range, the base equilibrium is stable**
  - **Outside this range TERPSICHORE fails in the vacuum calculation**
    - **Failure in vacuum calculation is a well-known problem with the TERPSICHORE code for tight aspect ratio configurations**

NCSX Equilibrium Scaled to ARIES Compact Stellarator: $\sqrt{\Phi}$ Contours



NCSX Equilibrium Scaled to ARIES Compact Stellarator: $\sqrt{\Phi}$ Contours



GENERAL ATOMICS

SEQUENCE OF FIXED BOUNDARY HIGHER □ EQUILIBRIA CONSTRUCTED FROM VMEC BY UNIFORMLY SCALING PRESSURE

- Volume, Average major and minor radius and vacuum field held fixed:
  - □ Magnetic axis at a single toroidal plane) shifts outward as □ increases
- Equilibria tested for ideal n=1 mode family stability:
  - □ Squared growth rate □^{2} vs □: □^{2} > 0 □ stable / □^{2} < 0 □ unstable

□ / Wall position	1.7	2.0	2.5
4.1%	+8.65 x 10^{-5}	+8.65 x 10^{-5}	+8.65 x 10^{-5}
5.7%	+1.81 x 10^{-4}	+1.81 x 10^{-4}	+1.81 x 10^{-4}
7.0%	-8.66 x 10^{-4}	-9.47 x 10^{-3}	-1.62 x 10^{-2}
8.3%	-7.25 x 10^{-2}	-7.75 x 10^{-2}	-8.06 x 10^{-2}

□ □ limit □ 6% for intermediate wall of order twice minor radius

RELIABILITY OF VACUUM CALCULATION CAN BE IMPROVED FOR COMPACT STELLARATOR CONFIGURATIONS

- **Terpsichore vacuum calculation uses a pseudo-vacuum to describe the vacuum perturbed vector potential as a pseudo-displacement:**
  - **$\square A = \square_p \square B$**

- **For $a_w < 1.7$:**
  - **Analytically extract logarithmic singularity more carefully**
  - **Possibly resort to Greens Function method**

- **For $a_w > 2.7$:**
  - **Corrected logic in wall construction to eliminate crossover sections**
  - **Resort to alternative definitions of conformal wall:**
    - **Constant normal distance in toroidal plane projection**
    - **Constant normal distance in helical plane projection**

PROGRESS SUMMARY: JANUARY 2003 TO DECEMBER 2003

PLANNED TASK	STATUS	COMMENTS
Port Terpsichore	Completed February 2003	Terpsichore now running on GA Linux system (Lohan1) Benchmarked for LHD and QHS cases (with WA. Cooper)
Obtain Base VMEC Equilibrium	Completed March 2003	Obtained scaled NCSX equilibrium from PPPL to use as starting point (with Long-Poe Ku)
Stability Calculation using Terpsichore	Completed June 2003	Rewritten interface between VMEC and Terpsichore to use PPPL and GA VMEC and CRPP Terpsichore versions
Modify VMEC Equilibrium and check stability robustness	Completed October 2003	Reproduced PPPL scaled NCSX equilibrium using GA VMEC version and run sequence with increasing □
Free Boundary Equilibrium from PIES	Not Done: Insufficient resources	This should be done in the long run but is beyond present resources
Iterate Equilibrium and Stability	Not Done: Insufficient resources	This should be done in the long run but is beyond present resources

PROPOSED STABILITY ANALYSIS PLANS: 2004

TASK	STATUS	COMMENTS
Reconstruct base 3 field period equilibrium	Initiated December 2003	Reconstruct increasing □ equilibrium sequence from VMEC
Stability Calculation using Terpsichore	Not Yet Done	Test convergence, □ limit, and sensitivity
Reconstruct 2 field period equilibrium or other variant	Not Yet Done	Two-field period variant requires more input data to construct from VMEC.
Stability Calculation using Terpsichore	Not Yet Done	Test convergence, □ limit, and sensitivity
Fix Wall Construction for Close Walls	Not Yet Done	Extract logarithmic singularity carefully
Fix Wall Construction for Distant Walls	Not Yet Done	Wall construction in constant □ planes or other options

□ Is this the best approach for determining ARIES Stability limits?

RELEVANCE OF IDEAL MHD β LIMITS IN STELLARATORS IS NOT WELL UNDERSTOOD

- Historically, tokamaks and stellarators have been designed using ideal MHD stability criteria:
  - Ideal localized Mercier and ballooning criteria and global stability
- In tokamaks these limits are considered well understood:
  - Ideal MHD appears to predict not just tokamak stability limits but also growth rates and mode structures in many situations
  - Fast, global instabilities identified with disruptions and β collapse
  - Localized and weakly growing instabilities identified with benign MHD activity: Edge Localized Modes (ELMs), Sawteeth, etc.
- Modern large stellarators however appear to violate these limits:
  - β appears to be limited by a soft limit of degrading confinement:

    β limits in the tokamak sense have not yet been observed
  - Predicted localized MHD limits are grossly violated in many cases:

    LHD and W7AS have exceeded predicted β limits by a factor two
  - Global limits also appear to be exceeded in more recent experiments

But stellarators and tokamaks have the same underlying physics based on Maxwell's Equations and Newtonian mechanics!

MAXIMUM β IN W7-AS AND LHD APPEARS TO BE LIMITED BY CHANGES IN CONFINEMENT AND NOT MHD ACTIVITY

- **Quiescent plasmas with $\beta > 3\%$ are routinely created in W7-AS:**
  - **Predictions from CAS-3D (C. Nuhrenberg) find global instabilities at intermediate β values even below 2%**
  - **Pressure driven MHD activity is sometimes observed but does not appear to limit β:**
    - **Dominant m/n = 2/1**
    - **Modes typically saturate at relatively harmless levels**
    - **Modes disappear at high β possibly due to inward shift of $\iota = 1/2$ surface**
- **Situation appears to be similar in LHD:**
  - **Pressure driven MHD activity does not prevent access to higher β:**
    - **m/n = 2/1 modes typically saturate at moderate β**
    - **Mode disappears for $\beta > 2.3\%$**
  - **Some correlation observed however between mode onset and predicted linear stability threshold**

THREE PROPOSALS TO STIMULATE DISCUSSION AND RESOLVE THIS ISSUE

- **Local MHD stability criteria appear to be irrelevant for stellarators:**
  - **Infinite n modes are coupled to lower n**
  - **Infinite n should be stabilized by non-ideal effects**
    - **Use finite n codes instead**
- **Global stability in tokamaks and stellarators may not be so different:**
  - **Tokamaks also routinely violate some MHD stability limits**
    - **MHD limits are open to interpretation and cannot be applied blindly as absolute hard limits**
    - **MHD limits can be sensitive to details in the equilibrium**
      - **Need to reconstruct discharge equilibria and understand nonlinear consequences of linear instabilities**
- **Equilibrium codes can provide some limited guarantees of stability:**
  - **An equilibrium computed under certain constraints must be stable unless those constraints can be avoided by a physically valid motion:**
  - **Otherwise any iterations for force balance in which an iterative error mimics an allowed perturbation will evolve away from the equilibrium unless constrained to not do so**
    - **Equilibrium codes can be considered stability codes**
    - **Subject to important caveats!**

MAJOR QUESTION:
SHOULD PREDICTIONS BASED ON NESTED FLUX SURFACE EQUILIBRIA BE IGNORED

- **Local stability criteria should probably be ignored:**
  - **There is little reason that infinite n should provide a physical limit**
  - **Finite n corrections appear to be large given the difference between the global code limits and the infinite n localized limits**
- **Global MHD stability is probably valid but must be applied to the right equilibrium:**
  - **Need to use the measured equilibrium profiles**
  - **May need to construct a non-nested flux surface equilibrium (with islands)**
  - **Flux surfaces might not even exist**
- **The nonlinear consequences are crucial in interpreting the results of a stability calculation:**
  - **Generally it might be expected that internal modes surrounded by a fairly robust and stable outer shell might be benign**
  - **Is there a way to quantify this without doing the full nonlinear calculation?**

WHAT SHOULD WE DO?
HOW SHOULD WE PROCEED?

- **Is there a role for nested flux surface equilibrium and stability codes?**
  - **Under what conditions is nested surfaces a valid approximation for stability calculations?**
  - **Does linear instability of a nested flux surface equilibrium simply result in benign nonlinear evolution to a 'nearby' non-nested state?**

Resolution requires testing global MHD stability using actual discharge equilibria

- **Detailed measurements of stellarator and pressure profiles are critically needed**

Only then can we decide if MHD calculations with nested surfaces are useful

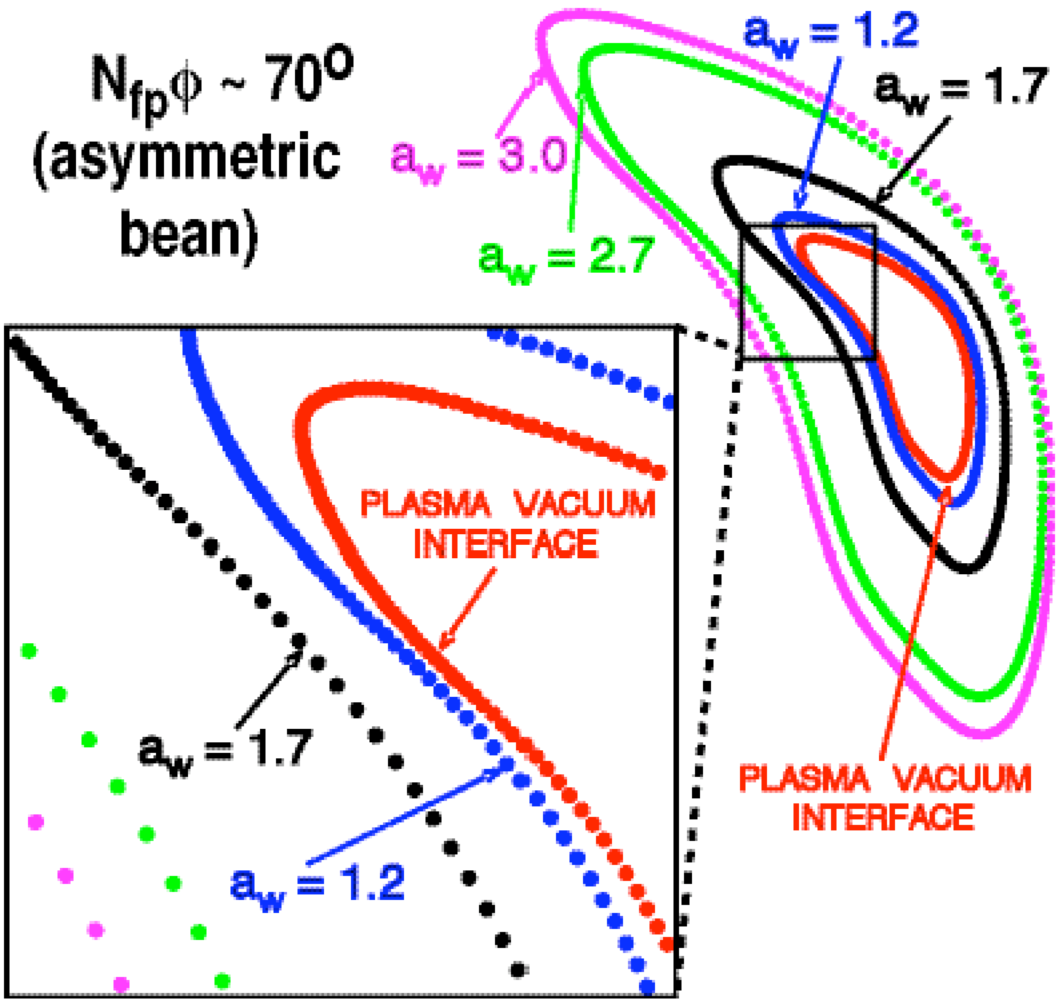
  - **Compact Stellarators are probably more tokamak like than conventional stellarators!**

- **If nested surfaces are not valid, can the stability problem be formulated in terms of finding nonlinearly stable equilibria?:**
  - **Is it possible to develop a general equilibrium code with few imposed constraints that can guarantee stability?**
  - **How can one distinguish a failure of the numerical scheme to converge from nonexistence of a stable nearby equilibrium?**

ASSESSMENT OF β LIMITS FOR COMPACT STELLARATORS:

BACKUP AND SUPPORTING MATERIALS

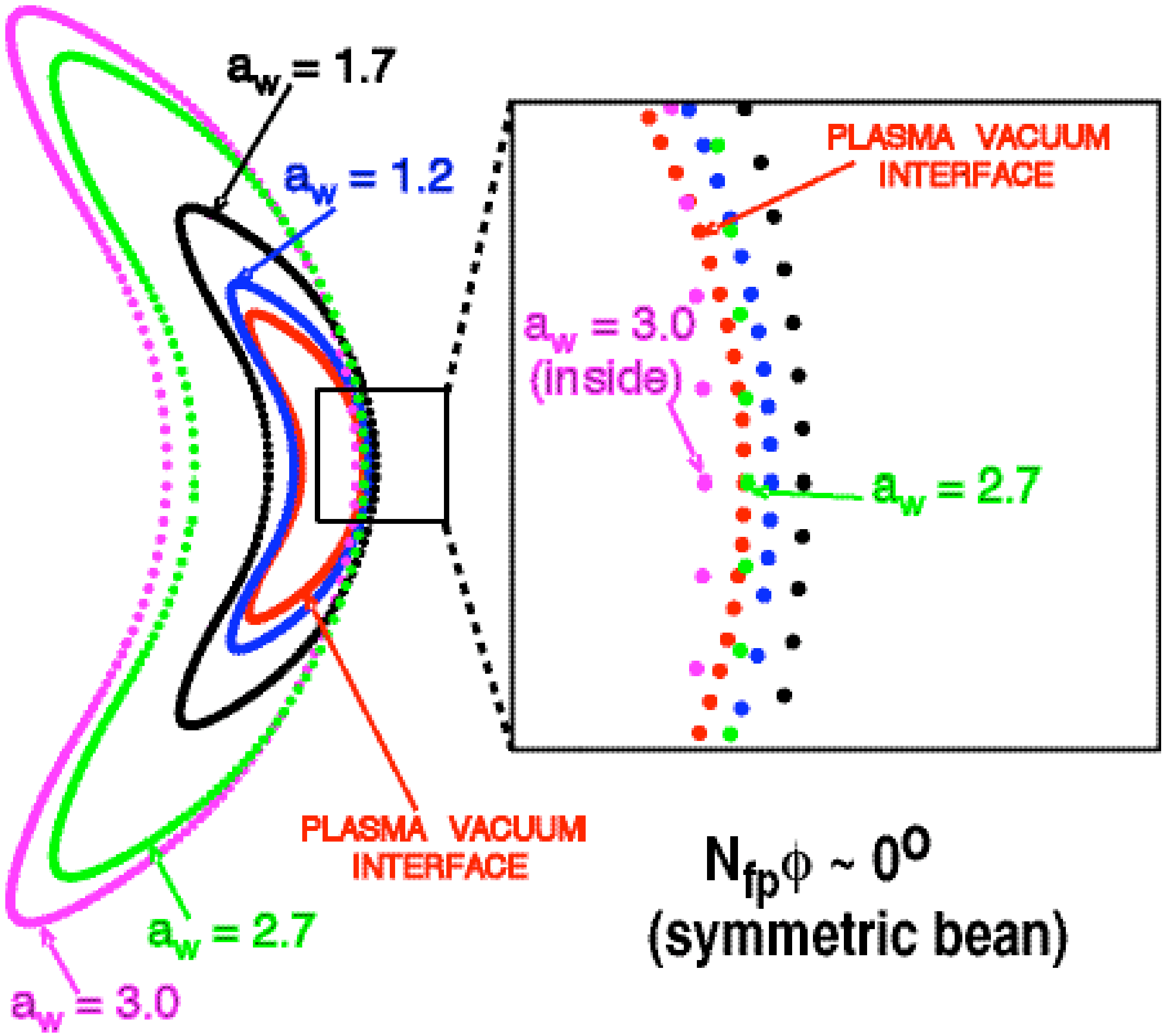
CAUSE OF FAILURE IN STABILITY CALCULATION ANALYZED AND IDENTIFIED



For a_w < 1.7:
Wall approaches plasma surface at a point on inboard side at one toroidal plane

Vacuum calculation fails due to logarithmic singularity as plasma-wall distance locally vanishes

CAUSE OF FAILURE IN STABILITY CALCULATION ANALYZED AND IDENTIFIED



Increasing a_w:
Outboard wall begins to move inward slightly at toroidal planes where plasma is bean shaped and up-down symmetric

For $a_w > 2.7$:
Wall cuts plasma-vacuum interface at point on outboard midplane

DISCUSSION POINT #1:
LOCAL MHD STABILITY CRITERIA APPEAR TO BE IRRELEVANT FOR STELLARATORS

- **Localized modes predicted to be unstable for β well below the global MHD limits should be stabilized by kinetic effects:**
  - **Finite n corrections are needed for physically meaningful predictions**
  - **In tokamaks, finite toroidal mode number n corrections to ballooning and Mercier stability are generally small**
    - **The infinite n calculation accurately reflects the real limit**
  - **In stellarators, the global stability codes in principle incorporate the high n localized modes with low and intermediate n**
    - **In practice the high n modes are numerically excluded**
- **In tokamaks high and low n are uncoupled and evaluated separately:**
  - **In Stellarators, they are coupled in principle and this is not accounted for in the localized criteria**
- **It is more realistic to ignore localized Mercier and ballooning limits in Stellarators and just use low and intermediate n global calculations:**
  - **By excluding the high n modes that in practice are stabilized by finite orbit effects the global codes are more closely reflecting the physics**
  - **In the global calculations the range of n needs to be terminated at the limit where finite orbit effects become important**

DISCUSSION POINT #2:

THE SITUATIONS REGARDING GLOBAL STABILITY ARE NOT ALL THAT DIFFERENT

- **Tokamaks also routinely violate some MHD stability limits:**
  - **MHD limits are open to interpretation and cannot be applied blindly as absolute hard limits**
  - **MHD limits can be sensitive to details in the equilibrium**

- **There are also some important distinctions between tokamaks and stellarators that may produce superficially different behavior**
  - **MHD theory, as applied to both, assumes the existence of nested flux surfaces:**
    - **In tokamaks this is sometimes not the case but normally it is an accurate assumption**
    - **In stellarators this is not always the case:**

Surfaces may not exist !

They may exist but be non-nested !

  - **We already know this to be partly true! But:**
    - **Given the sensitivity of the stability to the equilibrium the assumption of nested flux surfaces might be a poor approximation for stability even if islands are small**

TOKAMAKS ALSO ROUTINELY VIOLATE SOME MHD STABILITY LIMITS

- **The most well known example is the internal kink instability:**
  - **Tokamaks routinely operate with $q < 1$**
  - **The sawtooth instability is a consequence of the internal kink but is not at all well described by it**
    - **Non-ideal effects are important for low growth rate modes**
    - **Nonlinear consequences are usually nondisruptive**

- **Tokamaks also routinely violate Mercier interchange stability limits:**
  - **The Mercier limit is normally close to the internal kink limit but appears to be largely irrelevant in tokamaks**

- **Tokamak ballooning modes can have consequences near 'the β limit':**
  - **Interchange modes are in principle a special case of ballooning**
  - **Consequences of reaching ballooning limit are not always devastating**
    - **Soft β limit**

- **In H-mode Tokamaks also routinely reach intermediate n external mode stability limits:**
  - **ELMs appear to be the result of these instabilities**
    - **Nonlinear consequences are generally benign**

STABILITY LIMITS DEPEND SENSITIVELY ON THE EQUILIBRIUM

- **It is not normally sufficient to fit the equilibrium to just the global characteristics of tokamak discharges:**
  - **Stability depends quite sensitively on the details of both the current density (or safety factor) and pressure profiles**
    - **One can obtain widely varying results depending on the form assumed for the profiles for similar global parameters**
    - **Profiles need to be measured accurately and used in reconstructing the equilibrium for the stability calculations**

- **In Stellarators the equilibrium is believed to be known largely from the external coils: But**
  - **The □profile is often taken from the vacuum profile**
    - **It is not normally measured in the discharge and may be different at finite □**
  - **The pressure profile is not known as a function of flux**
    - **At most it is measured as a function of space and the mapping to flux space needed for the equilibrium depends on the □profile**

ASSUMPTION OF NESTED FLUX SURFACES MAY NOT BE VALID FOR LINEAR STABILITY

- **The assumption of nested flux surfaces may be invalid:**
  - **At least it may be an insufficiently good approximation to yield the observed stability**
  - **Finite □ can deteriorate the nested vacuum surfaces and given the sensitivity of the stability to the equilibrium configuration**
    - **Stability predictions using nested surfaces could be meaningless at finite □**

- **The islands and stochastic regions may be small but they may be ubiquitous throughout significant regions of the cross section:**
  - **Local flattening of the profiles and non-nested topology may yield very different stability from the ‘nearby’ nested configuration**
    - **The nested configuration may be linearly unstable but evolve nonlinearly to a configuration with ‘braided’ surfaces or thin islands, with flattened profiles in these regions**
  - **The new configuration will be linearly stable**
    - **The linear stability calculation using the approximate nearby nested flux surface equilibrium will yield the wrong result!**

DISCUSSION POINT #3:
EQUILIBRIUM CODES CAN PROVIDE SOME LIMITED GUARANTEES OF STABILITY

- **Equilibrium, stability and transport are not separable in stellarators:**
  - **Existence of a nested flux surface equilibrium can be considered as either an equilibrium or a stability problem:**
    - **Unstable equilibria with nested surfaces will evolve to a nearby non-nested surface state lower energy if physically possible**
  - **Transport is strongly dependent on underlying equilibrium magnetic topology and in turn determines the possible equilibrium profiles**

- **Equilibrium codes can be considered stability codes:**
  - **An equilibrium computed under certain constraints must be stable unless those constraints can be avoided by a physically valid motion:**
  - **Otherwise any iterations for force balance in which an iterative error mimics an allowed perturbation will evolve away from the equilibrium unless constrained to not do so**
  - **A variational code will find the energy minimizing state unless constrained to not do so**

DIFFERENT EQUILIBRIUM CODES GUARANTEE VARYING DEGREES OF STABILITY

- VMEC imposes simply nested flux surfaces:
  - □ Profiles assumed for p(□) and a function specifying current density j
    - □ Equilibria should be stable to all topology preserving and profile preserving (i.e. fixed p(□) and j) MHD instabilities
  - □ Can be unstable to perturbations that change topology or any other variable that was constrained in the equilibrium calculation

- Free boundary direct equilibrium codes PIES and HINST have fewer constraints on the equilibrium:
  - □ Fewer constraints □ more stability guarantees
  - □ Topology is not constrained
  - □ Profiles assumed for p(□) and a function specifying the current density j (an integration constant on each flux contour for PIES)
    - □ Equilibria should be stable to all profile preserving (i.e. fixed p(□) and j) MHD instabilities

Even instabilities that do not preserve topology
i.e. that create or destroy islands!

GUARANTEE OF STABILITY IS SUBJECT TO IMPORTANT CAVEATS

- **Claim is that convergence to physically unstable equilibria is not possible unless constraints are imposed on the numerical procedure that prevent either:**
  - **Equilibrium states without specific symmetries (eg. axisymmetric), or**
  - **Symmetry breaking perturbations away from force balance**

> **Lack of convergence does not imply lack of stable equilibrium!**
>
> **Only the converse is claimed: that lack of stability will prevent convergence unless constraints are imposed!**

- **Free boundary direct equilibrium codes assume p = constant for flux surfaces inside islands:**
  - **Pressure is a different function of flux in separate simply connected regions**
    - **p is not a *single valued* function of □**
  - **States with different prescriptions for the multiple values for p and j in different simply connected regions (islands etc.) are possible and may be physically accessible**
  - **The actual profiles will be determined by transport and the topology of the region**