IDEAL MHD BETA LIMITS FOR ARIES-CS

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Contributions to this work from several colleagues are greatly appreciated:

W.A. Cooper (CRPP/EPFL) for aid in porting TERPSICHORE to local systems, aid in running it, and consulting

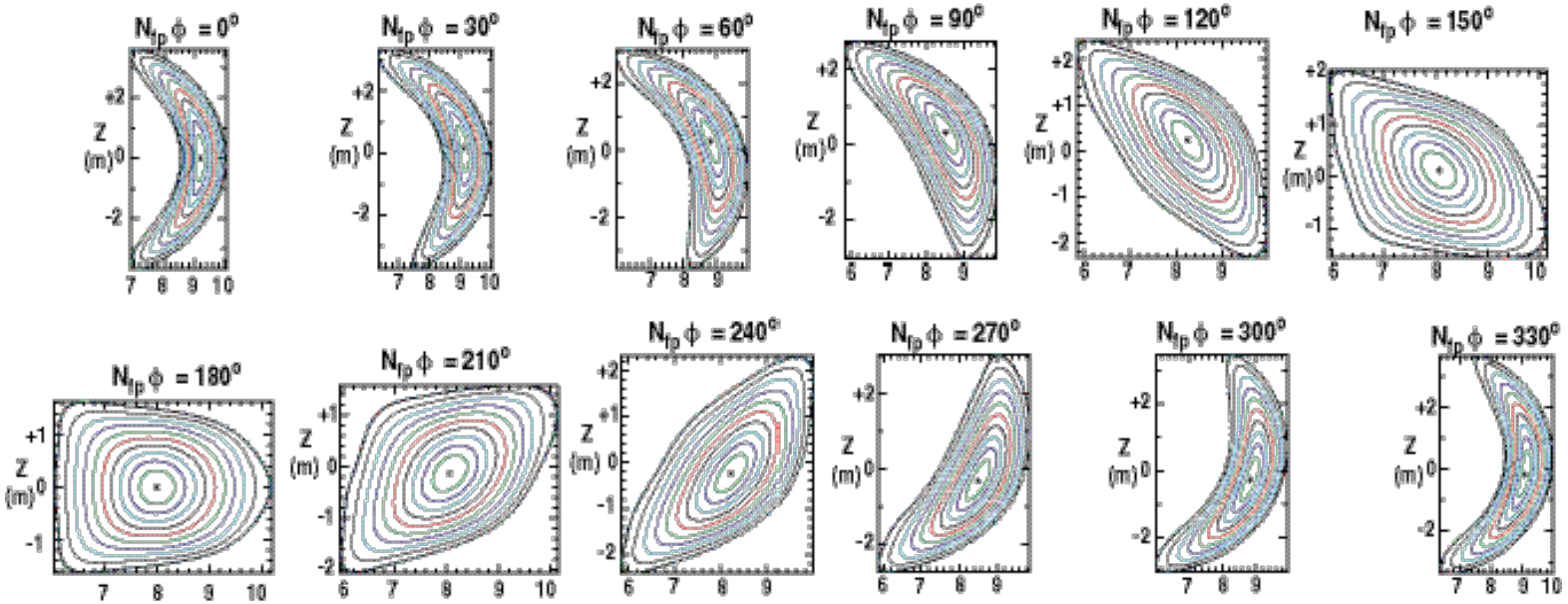
Long-Poe Ku (PPPL) for providing initial ARIES Equilibrium

ORIGINAL STABILITY ANALYSIS PLANS: 2003

- **Port TERPSICHORE to local GA Linux workstation:**
  - Assistance from W.A. Cooper with non source code libraries
  - Assistance from G.Y. Fu (PPPL) with interpretations
- **Obtain equilibrium from VMEC (M.C. Zarnstorff PPPL):**
  - Stability analysis using TERPSICHORE for significant range of toroidal mode number n
  - Monitor high n stability but ignore for optimization
  - Increase β or estimate modifications to plasma parameters required to stabilize
- **Modify VMEC equilibrium accordingly (M.C. Zarnstorff):**
  - Iterate with stability analysis using TERPSICHORE
  - Converge on final equilibrium when β limit is reached and simple adjustments to other parameters are ineffective
- **Reconstruct free boundary equilibrium using PIES (M.C. Zarnstorff):**
  - Modification to coils and plasma parameters to reproduce a reasonable set of free boundary nested surfaces
- **Iterate with VMEC and TERPSICHORE calculations:**
  - Linear resistive and nonlinear MHD calculation of final equilibria

COMPACT STELLARATOR EQUILIBRIUM AND STABILITY ANALYSIS

- **Scaled-up NCSX equilibrium (Long-Poe Ku PPPL) reproduced using the GA version of the VMEC code:**
  - **Equilibrium <β> = 4.1% and A = 4.47**
  - **Equilibrium toroidal flux contours for a series of toroidal angles covering a single field period**



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COMPACT STELLARATOR STABILITY ANALYSIS

- **Stability results obtained for the three-period scaled up NCSX equilibrium**
  - **TERPSICHORE preprocessor modified to accept input from the latest version of VMEC in use at PPPL and GA**
  - **TERPSICHORE results using the PPPL and GA equilibria are essentially identical**

- **Stability results for the scaled-up equilibrium restricted to a range of moderately placed external conformal conducting walls**
  - **Between 1.7 and 2.7 times the minor radius of the plasma**
  - **For the wall in this range, the base equilibrium is stable**
  - **Outside this range TERPSICHORE fails in the vacuum calculation**

SEQUENCE OF HIGHER □ EQUILIBRIA CONSTRUCTED FROM VMEC

- **Volume, Average major radius, average minor radius, and vacuum toroidal field held fixed**
  - **Magnetic axis shift outward as □ increases**

<□>(%)	4.1	4.9	5.7	7.0	8.3
$R_m(0)$ (m)	9.20	9.21	9.22	9.24	9.26
$V(m^3)$	556.5	556.5	556.5	556.5	556.5
R(m)	8.25	8.25	8.25	8.25	8.25
a(m)	1.85	1.85	1.85	1.85	1.85
A	4.47	4.47	4.47	4.47	4.47
<B>(T)	5.46	5.46	5.46	5.46	5.46

EQUILIBRIA TESTED FOR IDEAL STABILITY AT INTERMEDIATE WALL POSITIONS

- **Squared growth rate γ^2 versus equilibrium β:**
  - **Tested for n=1 stability: Positive γ^2 $\Rightarrow$ stable/Negative γ^2 $\Rightarrow$ unstable**

β / Wall position	1.7	2.0	2.5
4.1%	$+8.65 \times 10^{-5}$	$+8.65 \times 10^{-5}$	$+8.65 \times 10^{-5}$
5.7%	$+1.81 \times 10^{-4}$	$+1.81 \times 10^{-4}$	$+1.81 \times 10^{-4}$
7.0%	-8.66×10^{-4}	-9.47×10^{-3}	-1.62×10^{-2}
8.3%	-7.25×10^{-2}	-7.75×10^{-2}	-8.06×10^{-2}

$\Rightarrow$ β limit $\approx$ 6% for intermediate wall of order twice minor radius

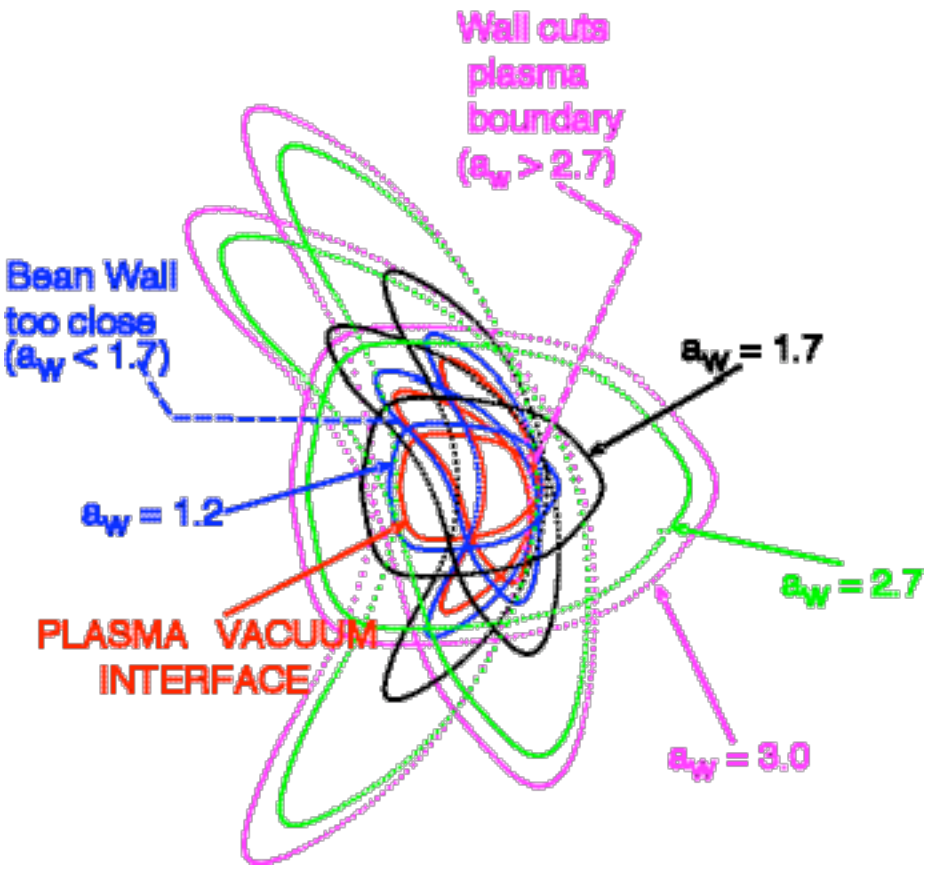
- **Continuing studies to confirm this limit:**
  - **Convergence with mesh size**
  - **Other toroidal mode families**

CAUSE OF FAILURE IN STABILITY CALCULATION ANALYZED

- **TERPSICHORE analysis tools from obtained from Lausanne group:**
  - **Tools to analyze the vacuum and wall construction**
- **Cause of TERPSICHORE vacuum failures determined:**
  - **$a_w < 1.7$:**
    - **wall approaches plasma surface at a point on inboard side at one toroidal plane**
    - **vacuum calculation fails due to logarithmic singularity as plasma-wall distance locally vanishes**
  - **Increasing a_w:**
    - **outboard wall begins to move inward slightly at toroidal planes where plasma is bean shaped and up-down symmetric**
  - **$a_w > 2.7$:**
    - **wall cuts plasma-vacuum interface at point on outboard mid-plane**
- **This is a well-known problem with the TERPSICHORE code:**
  - **Source of the comment in the earlier evaluation of the numerical equilibrium and stability tools**

"A completely robust free boundary and vacuum formulation does not yet exist even though TERPSICHORE does have a working vacuum boundary condition for some cases at least. This may be an issue for the ARIES study and some development may be required to obtain this capability fully."

THIS ISSUE WILL TAKE A SIGNIFICANT EFFORT TO RESOLVE FOR COMPACT STELLARATORS



Constructed wall positions for varying wall expansion parameters a_w, at 3 toroidal planes:

N_{fp} □ ~ 0° (symmetric bean)

N_{fp} □ ~ 70° (asymmetric bean)

N_{fp}□ ~ 180° (symmetric, Flat Dee)

boundary: red curves

a_w = 1.7: blue and black

a_w = 2.7: green and violet

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OTHER DEVELOPMENTS: ROLE OF IDEAL MHD STABILITY IN STELLARATORS

- A special discussion of the role of ideal stability calculations scheduled for the National Stellarator Theory Teleconferences:
  - M. Zarnstorff presented the results he showed at the ARIES May 7 team Meeting (June teleconference)
  - Alan Turnbull and Paul Garabedian presented their views (July teleconference)
  - Posted at the National Stellarator Theory Teleconference website: http://www.pppl.gov/ncsx/Physics/NSTT/Theory_Index.html
- Results can be summarized as:
  - Local stability criteria should probably be ignored:
  - Global MHD stability is probably valid but must be applied to the correct equilibrium to obtain meaningful predictions
  - Equilibria being tested theoretically for global stability may be inadequate on an even more basic topological level
  - Nonlinear consequences are crucial in interpreting stability results
- Consequence is that it is not presently clear how valuable linear MHD stability predictions using nested flux surface equilibria really are:
  - They might, in fact, be quite misleading
  - It may be necessary to use free boundary non-nested equilibria and/or nonlinear stability studies to obtain meaningful predictions

SUMMARY OF VIEWS PRESENTED TO NATIONAL STELLARATOR THEORY TELECONFERENCE

- **Local stability criteria should be ignored:**
  - **There is little reason that infinite n should provide a physical limit:**
    - **> Finite n corrections appear to be large**
  - **Reasonable theoretical justifications exist for ignoring infinite n limit**
- **Global MHD stability is probably valid but must be applied to the correct equilibrium to obtain meaningful predictions:**
  - **Tokamak stability experience need measured equilibrium profiles**
  - **Stellarator experiments do not normally measure the profile:**
    - **> Normally guess from theoretical arguments**
    - **> Similar arguments for tokamak stabilitywere found to be unreliable once actual measurements became available**
- **Equilibria may be inadequate on an even more basic topological level:**
  - **Assumption of nested flux surfaces may be invalid for testing stability**
    - **> Test stability of non-nested flux surface equilibrium with islands**
- **Nonlinear consequences are crucial in interpreting stability results:**
  - **Generally it might be expected that internal modes surrounded by a fairly robust and stable outer shell might be benign**
  - **This is a reasonable heuristic assumption**
    - **> Need a way to quantify this without the full nonlinear calculation**

OTHER DEVELOPMENTS: CONSTRUCTION OF A STELLARATOR EQUILIBRIUM FITTING CODE

- A 3D equilibrium fitting code is needed to resolve the problem of insufficiently well known discharge equilibria in present experiments
- Development of a 3D equilibrium reconstruction code progressing in a collaboration between L.L. Lao of General Atomics, S.P. Hirshman and E.A. Lazarus of ORNL, and J.D. Hanson and S.F. Knowlton of Auburn University
  - Two codes V3RFUN and V3POST to compute developed to compute 3D response functions and evaluate magnetic signals for various stellarator equilibria
    - > Applied to support design of magnetic diagnostics for NCSX and CTH compact stellarator devices
  - Implemented a surface approach to compute plasma contribution to the external magnetic diagnostics in V3POST
    - > Results of a convergence study indicate surface method is more sensitive than volume approach to number of poloidal grid points used in the integration

OTHER DEVELOPMENTS: STATUS OF 3D STELLARATOR LINEAR RESISTIVE MHD CODE

- **Current status of SPECTOR-3D: (Ben McMillan, ANU/Flinders U.)**
  - Can reproduce 3D ideal MHD instabilities:
    - > Mainly LHD and heliotron-E test cases
    - > Toroidal and poloidal coupling seems to be correct
  - Sensible results obtained for resistive interchange:
    - > Needs more testing
  - Resistive tearing modes can be resolved in some cases:
    - > 1D is fine
    - > Not yet robust in 2D and 3D
  - Cannot yet resolve low frequency tearing instabilities in 2D and 3D:
    - > May be a 'spectral pollution' problem associated with spurious numerical coupling of fast modes with finite resistivity
- **Spector3D uses VMEC input:**
  - Possible issues near the rational surfaces with possibly singular equilibrium quantities
  - VMEC also does not do well near the magnetic axis (but apparently there is a new version that fixes this)
- **Status summarized as:**
  - 'Not quite ready'

PROGRESS SUMMARY: JANUARY 2003 TO SEPTEMBER 2003

TASK	STATUS	COMMENTS
Port Terpsichore	Completed February 2003	Terpsichore now running on GA Linux system (Lohan1) Benchmarked for LHD and QHS cases (with WA. Cooper)
Obtain Base VMEC Equilibrium	Completed March 2003	Obtained scaled NCSX equilibrium from PPPL to use as starting point (with Long-Poe Ku)
Stability Calculation using Terpsichore	Completed June 2003	Rewritten interface between VMEC and Terpsichore to use PPPL and GA VMEC and CRPP Terpsichore versions
Modify VMEC Equilibrium and check stability robustness	In Progress April - September 2003	Reproduced PPPL scaled NCSX equilibrium using GA VMEC version and run sequence with increasing □
Free Boundary Equilibrium from PIES	Not Done	This should be done in the long run but is beyond present resources
Iterate Equilibrium and Stability	Not Done	This should be done in the long run but is beyond present resources