Towards An Analytic Model for Beam Spot Size in Ballistic Neutralized Transport

Simon Yu, J. Barnard, W.M. Sharp,
D. Welch, D. Rose, C. Olson

ARIES Meeting, San Diego
1/10/2002

Beam Envelope Model

The propagation of a beam from beam port to target can be described by an envelope equation:

$$\frac{d^2R}{dz^2} = \frac{\varepsilon^2}{R^3} + \frac{U}{R}$$

Where R -beam radius
ε -beam emittance
U -Net perveance
z -distance

Existing Model in System Code

- Assume total neutralization (i.e. beam is force-free)
- Then U =0 ε=constant

$$\frac{d^2 R}{dz^2} = \frac{\varepsilon^2}{R^3}$$

- Solution:

$$R_t = \frac{\varepsilon}{R_o / L}$$

Where R_t = beam radius at target
R_o= beam radius at beam port
L = chamber radius

Generalization to incomplete neutralization

- Emittance growth ε(z)
- Nonzero net perveance U(z) ≠ 0

Approximate solution for R_t

$$R_t = \frac{\varepsilon_t}{\left[(R_o / L)^2 - 2U_t \log\left(\frac{R_o}{R_t}\right)\right]^{\frac{1}{2}}}$$

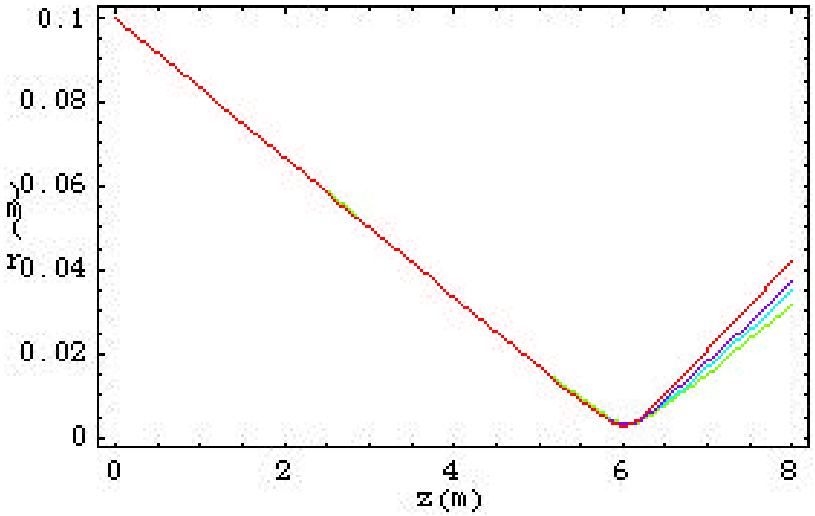
Observations:

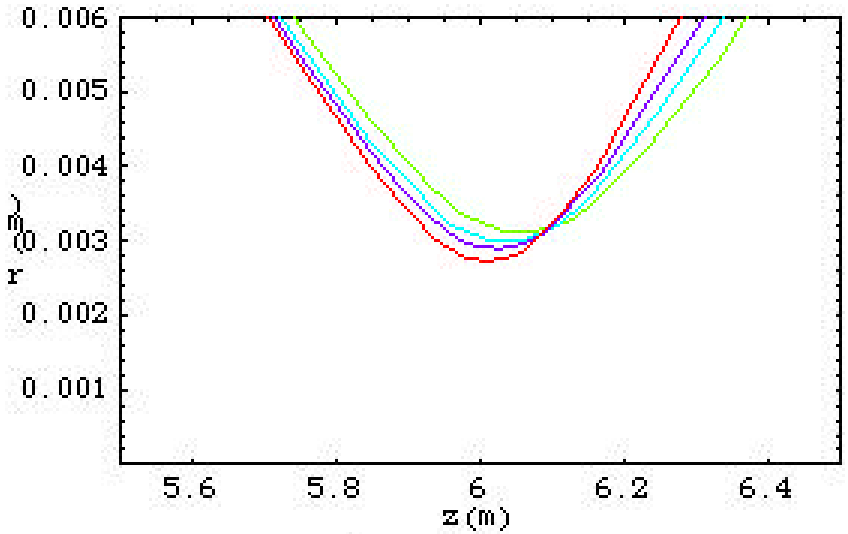
- R_t depends on final emittance ε_t and net perveance near the target U_t
- When space charge dominates

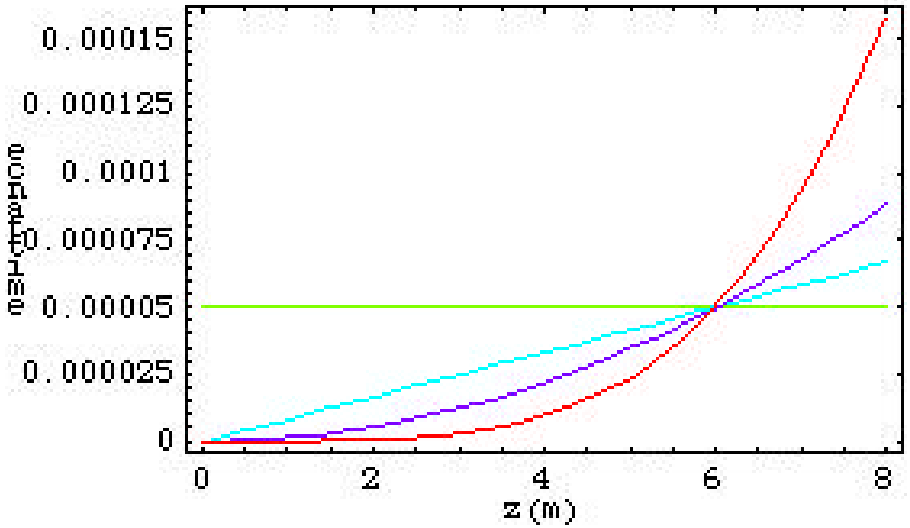
  $U_t < O$, $R_t \uparrow$
- When pinch forces dominate

  $U_t > O$, $R_t \downarrow$

Neck radius depends primarily on final emittance, not on the evolution of the emittance







To Calculate Emittance Growth

$$\frac{d^2\varepsilon^2}{dz^2} = \lambda^2 U^2 \quad \text{(exact)}$$

λ - dimensionless radial shape factor

$\lambda = 0$ for flat charge distribution

λ 0.5 for nearly charge neutralized beams

$U \sim Z\, m_e/m_i$

Final emittance given by (for U constant)

$$\boxed{\varepsilon_t^2 = \varepsilon_o^2 + \lambda^2 U^2 L^2}$$