Computational Modeling in Support of the Magnetic Intervention Concept

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HAPL Meeting
Princeton Plasma Physics Laboratory
December 12-13, 2006

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Outline

1. Description of EMHD model in cylindrical coordinates (based on model of D. Hewett)
2. Direct comparison of explicit PIC simulation to EMHD model
3. EMHD model of R. E. Pechacek magnetic intervention experiment
4. Preliminary EMHD modeling of “chamber-scale” magnetic intervention
5. Next steps…

1.) EMHD model implementation based on work of D. Hewett*

- We have implemented a version of D. Hewett's 2D cylindrical (r,z) field solver (advancing A_θ)
- Model includes "correct" evolution of vacuum magnetic fields
- Field solver implemented within Lsp code framework

*D. W. Hewett, J. Comp. Phys. **38**, 378 (1980)

Equation for A_θ

$$\frac{\partial A_\theta}{\partial t} - \frac{c^2}{4\pi\,\sigma}\frac{1}{}\nabla^2 \vec{A}\Big|_\theta + u_{er}\frac{1}{r}\frac{\partial\left(rA_\theta\right)}{\partial r} + u_{ez}\frac{\partial A_\theta}{\partial z} = 0$$

Conductivity is assumed to be a scalar to avoid carrying extra terms

B_θ can be obtained between ADI passes, but we have not implemented this yet.

In vacuum, this equation reduces to:

$$\nabla^2 \vec{A}\Big|_\theta = 0$$

Model Constraints:

- Most of the computational constraints associated with our previous EMHD solvers also apply here.
- In addition, a “grid-Reynolds” constraint applies, that when combined with the usual diffusion-rate constraint give the following inequality:

$$\left(\frac{c^2}{4\pi}\right)\frac{2\Delta t}{(\Delta x)^2} < \sigma\left(\mathrm{s}^{-1}\right) < \frac{2}{\nu\,\Delta x}\left(\frac{c^2}{4\pi}\right)$$

2.) Comparison of explicit EM PIC simulation and EMHD simulation:

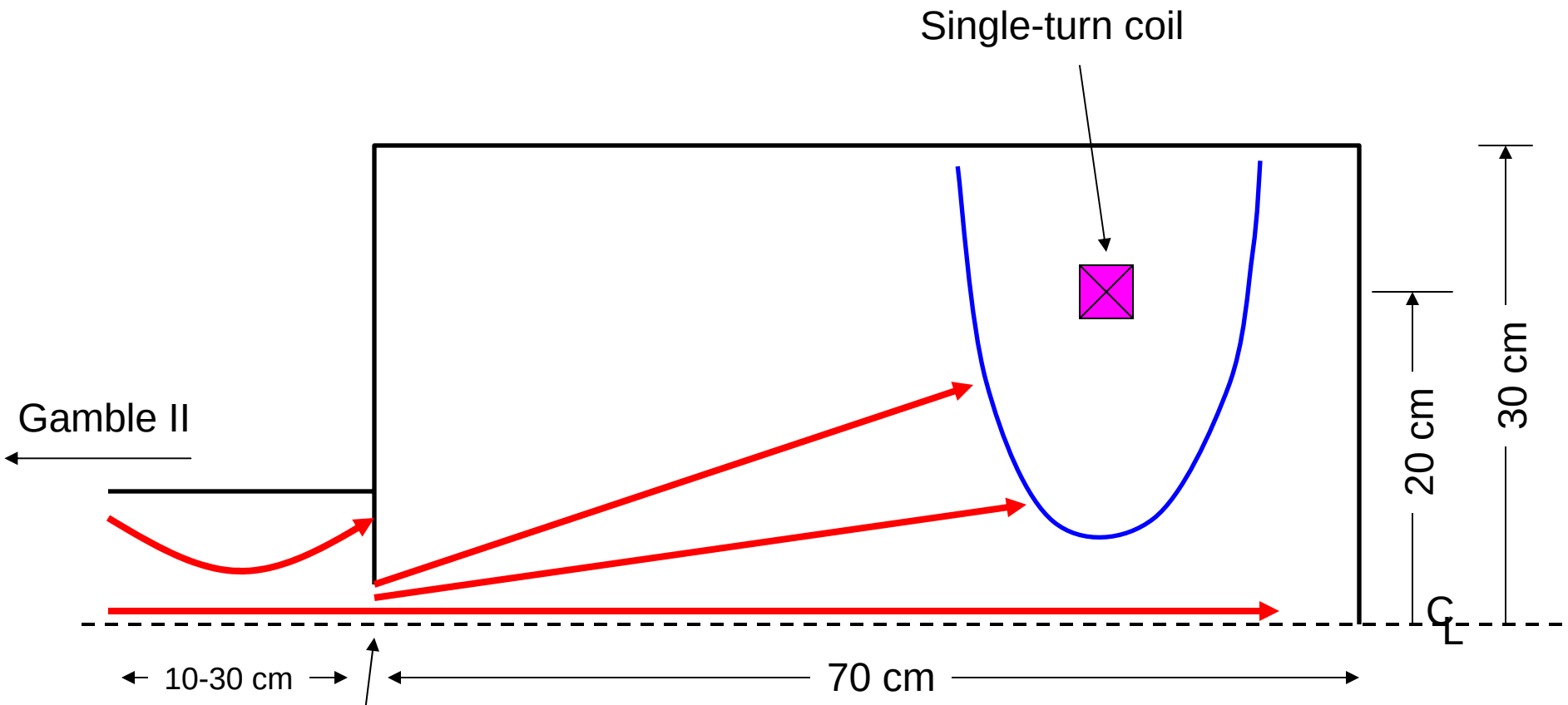
- Important benchmark of the EMHD model
- Assess impact of EMHD approximations on magnetic intervention modeling
- Explicit EM PIC uses
  - Inertial macro-particles for both ions and electrons
  - Complete set of Maxwell's equations on finite grid

The comparison is carried out using the simulation geometry for the (ill-fated) Gamble II experiment*:

- Explicit simulations for this problem geometry already completed.
- Relatively small-scale simulation problem for the EMHD algorithm (easily satisfying computational constraints for a relatively high conductivity)
- PIC simulations demonstrated diamagnetic penetration of a plasma into an applied magnetic field with a well defined electron sheath formed at the plasma/magnetic-field interface

*See D. V. Rose, *et. al* presentation, HAPL Meeting, ORNL, March 21, 2006

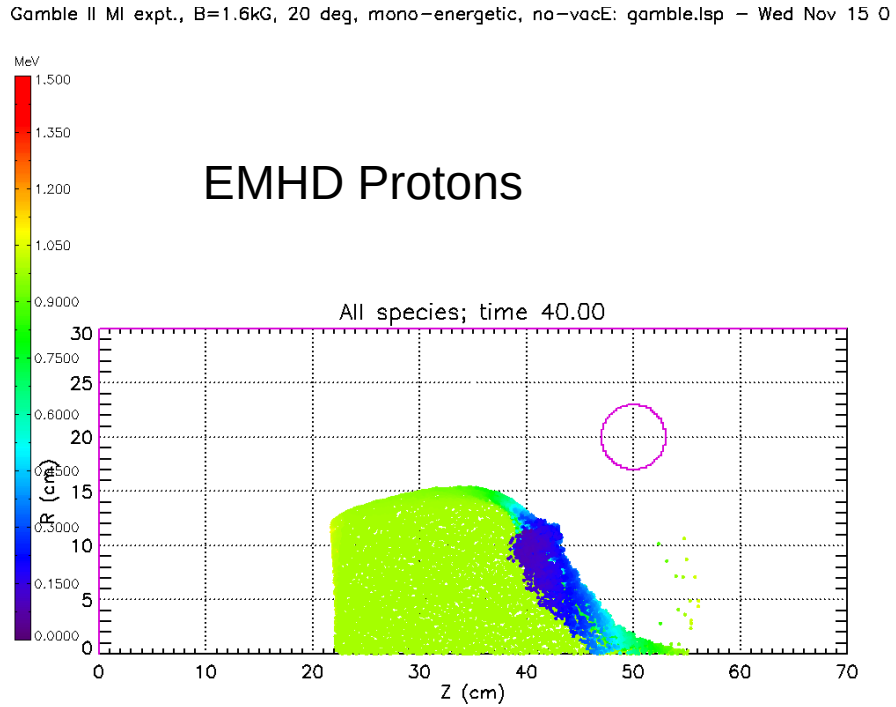
Gamble II Experiment: Schematic

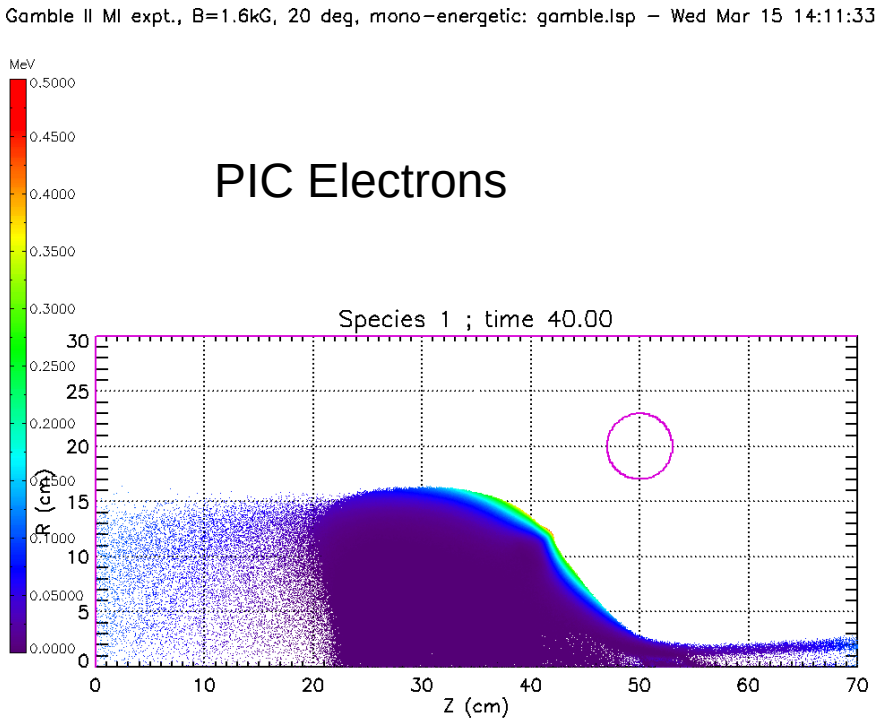


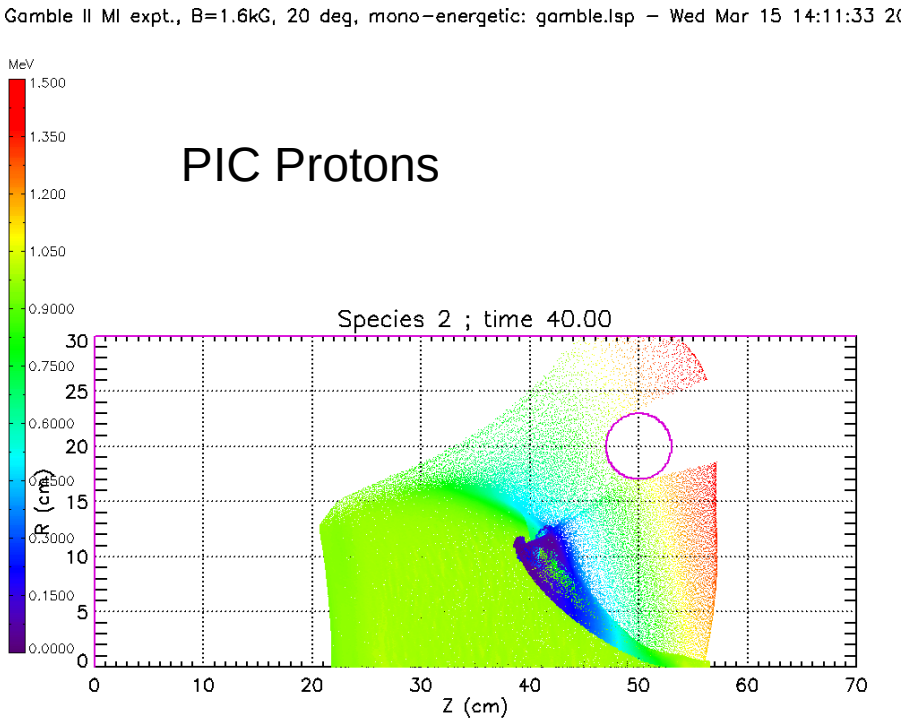
A portion of the total proton beam enters the chamber through an aperture.

Comparison: Proton dynamics at the sheath are essentially equivalent at 40 ns.

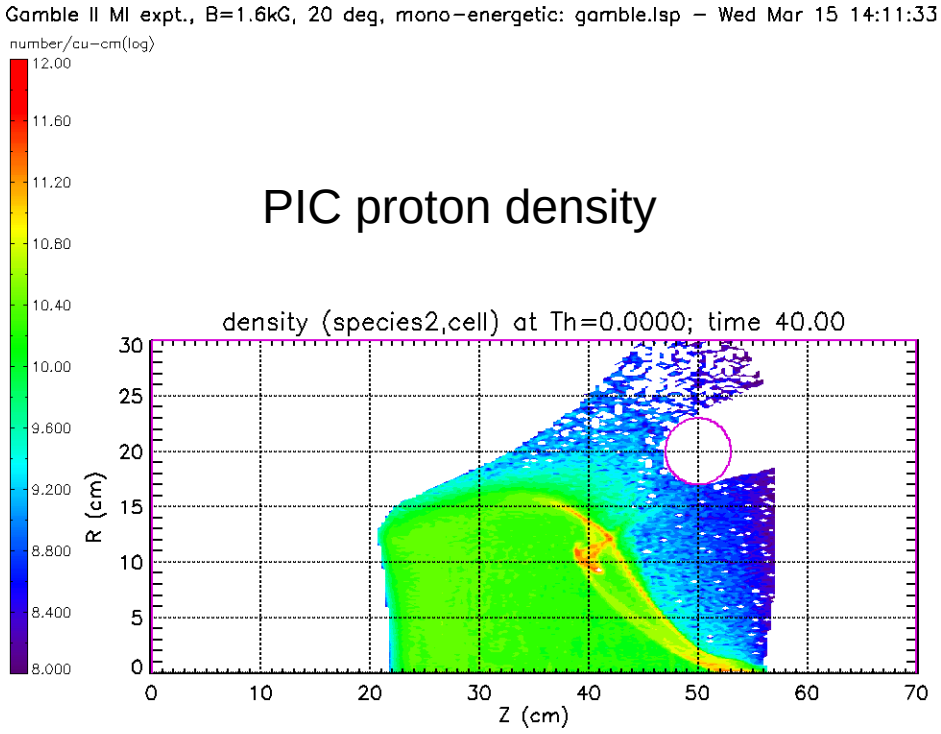
EMHD sim: ~15,600 particles (protons)
Explicit PIC sim: ~ 2,000,000 particles
(1,700,000 electrons, 300,000 protons)



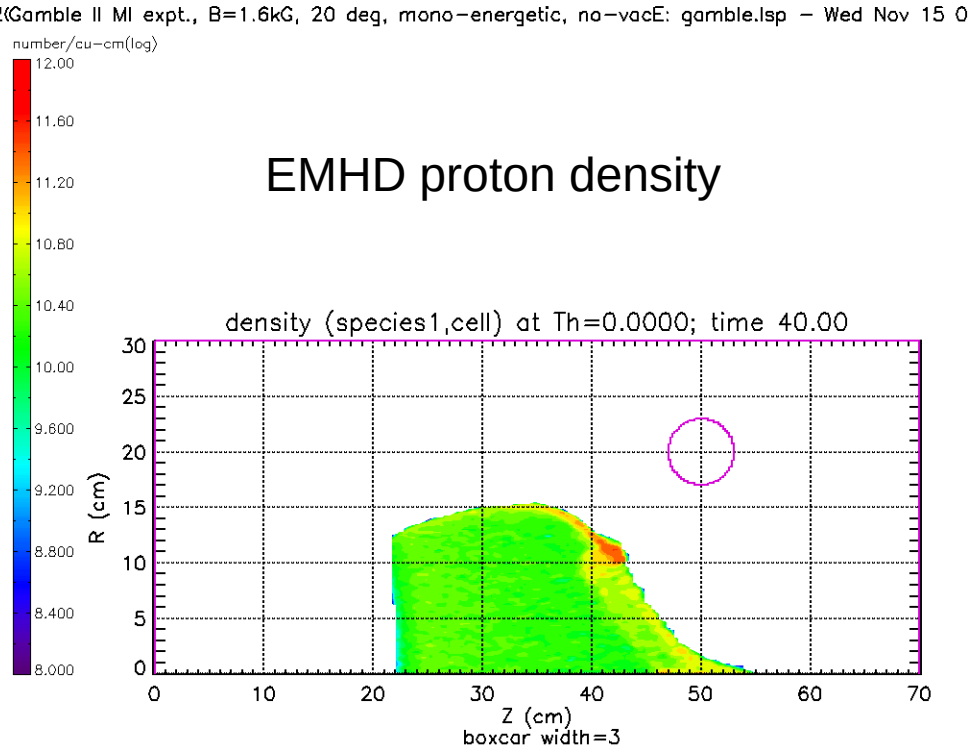




The proton density at 40 ns is remarkable similar: the density pile-up at (r,z)=(12,42) is even present, although less well resolved in the EMHD simulation.

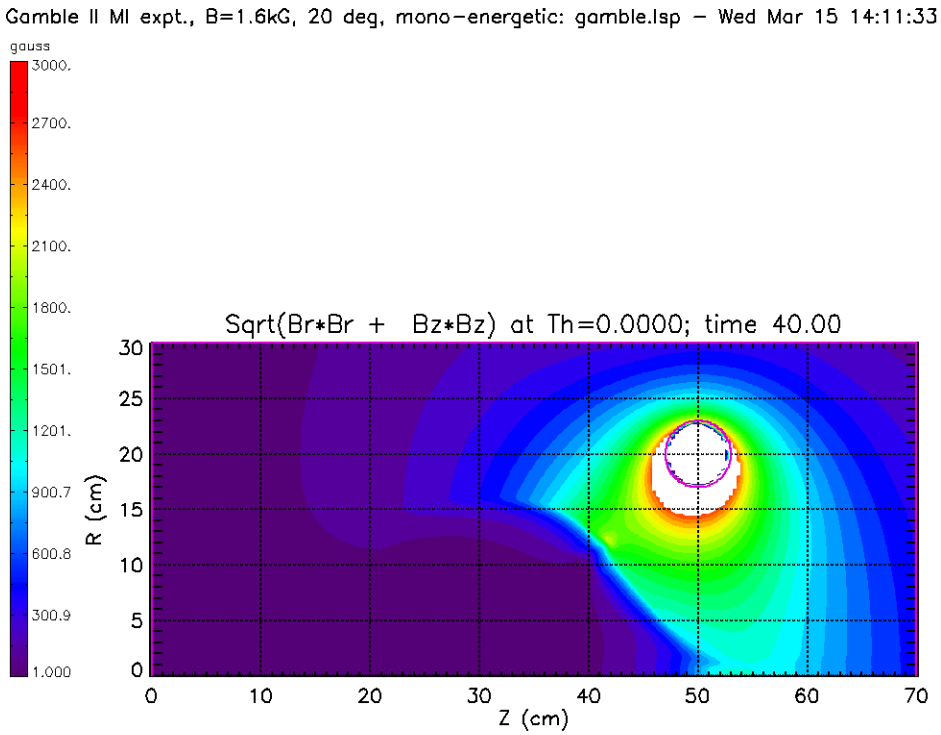


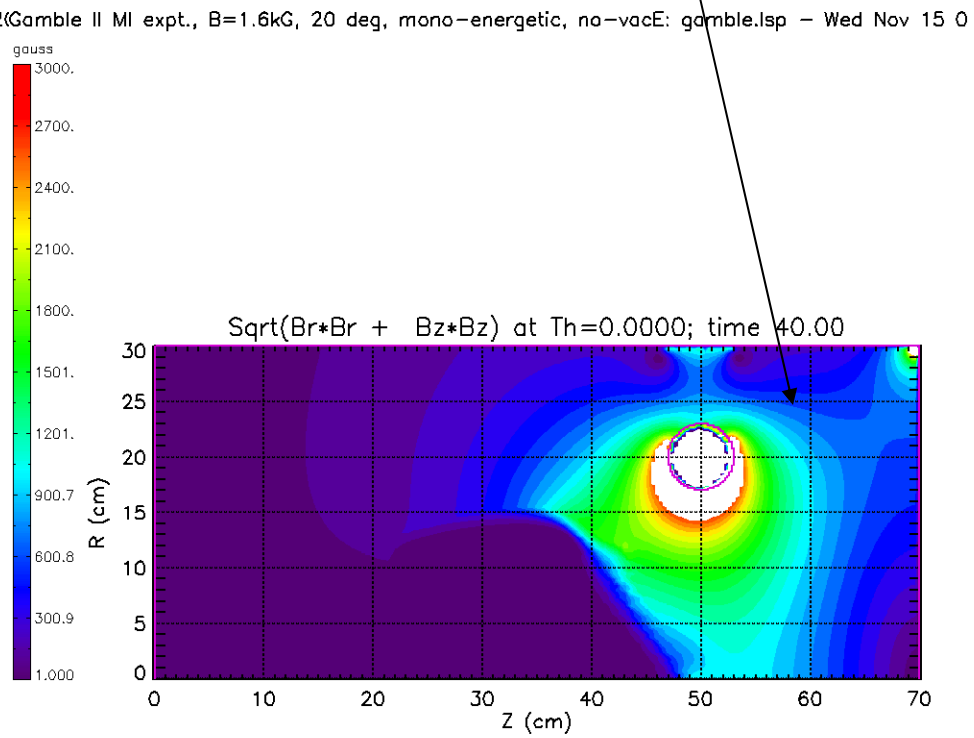
PIC proton density



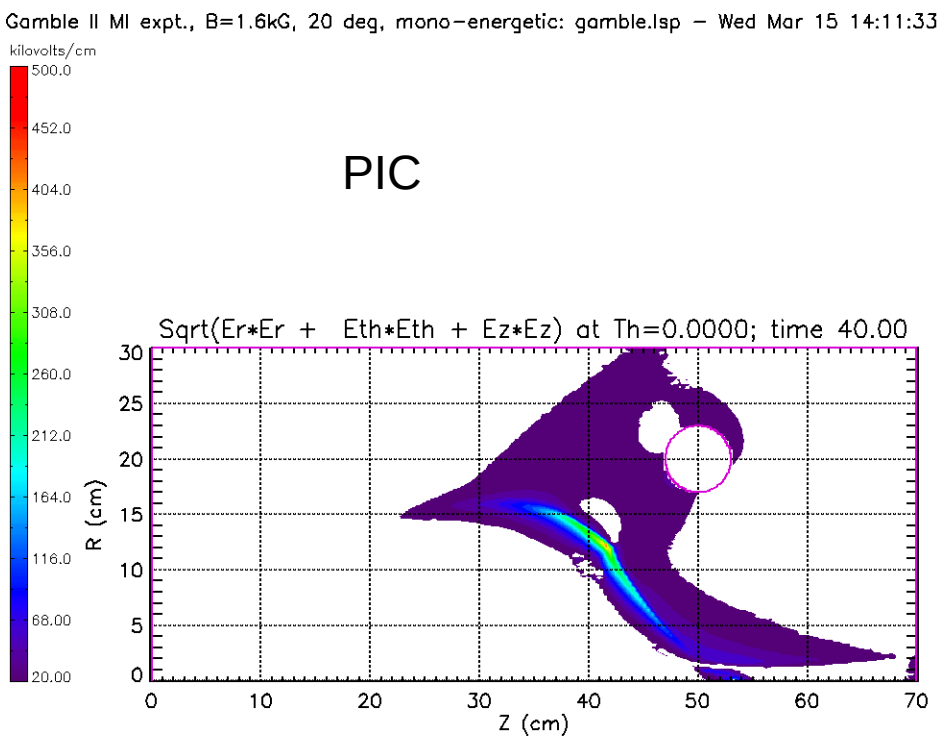
EMHD proton density

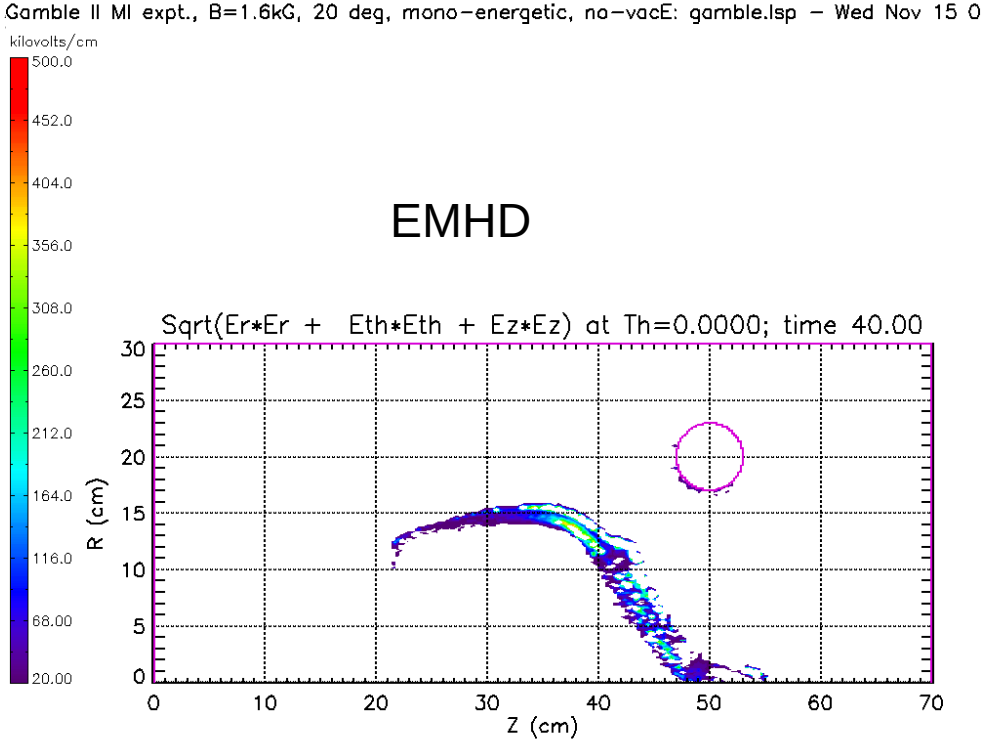
Contours of |B| in the vicinity of the sheath are in agreement. EMHD solution at large radius and on opposite side of coil is bad.





E-fields in vacuum region are huge (up to 10^4 kV/cm) and non-physical in the EMHD calculation by 40 ns. PIC shows E-fields confined to thin sheath region only.





3: Pechacek Experiment Modeling

- A two-stage laser system drives a 1-mm scale, solid D_2 pellet forming a plasma.
- The plasma is created inside the void of a cusp magnetic field.
- The adiabatically expanding plasma compresses the cusp field lines.
- Plasma ions escape from the “point” and “ring” cusps in the field geometry.
- Plasma ions are “deflected” away from the chamber walls

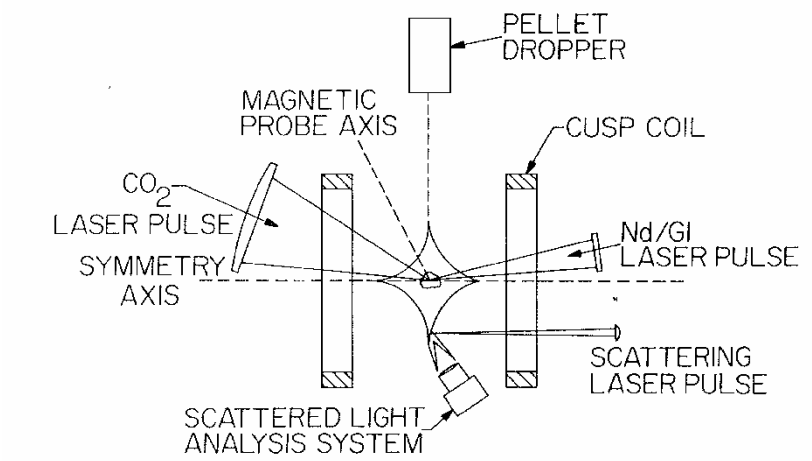
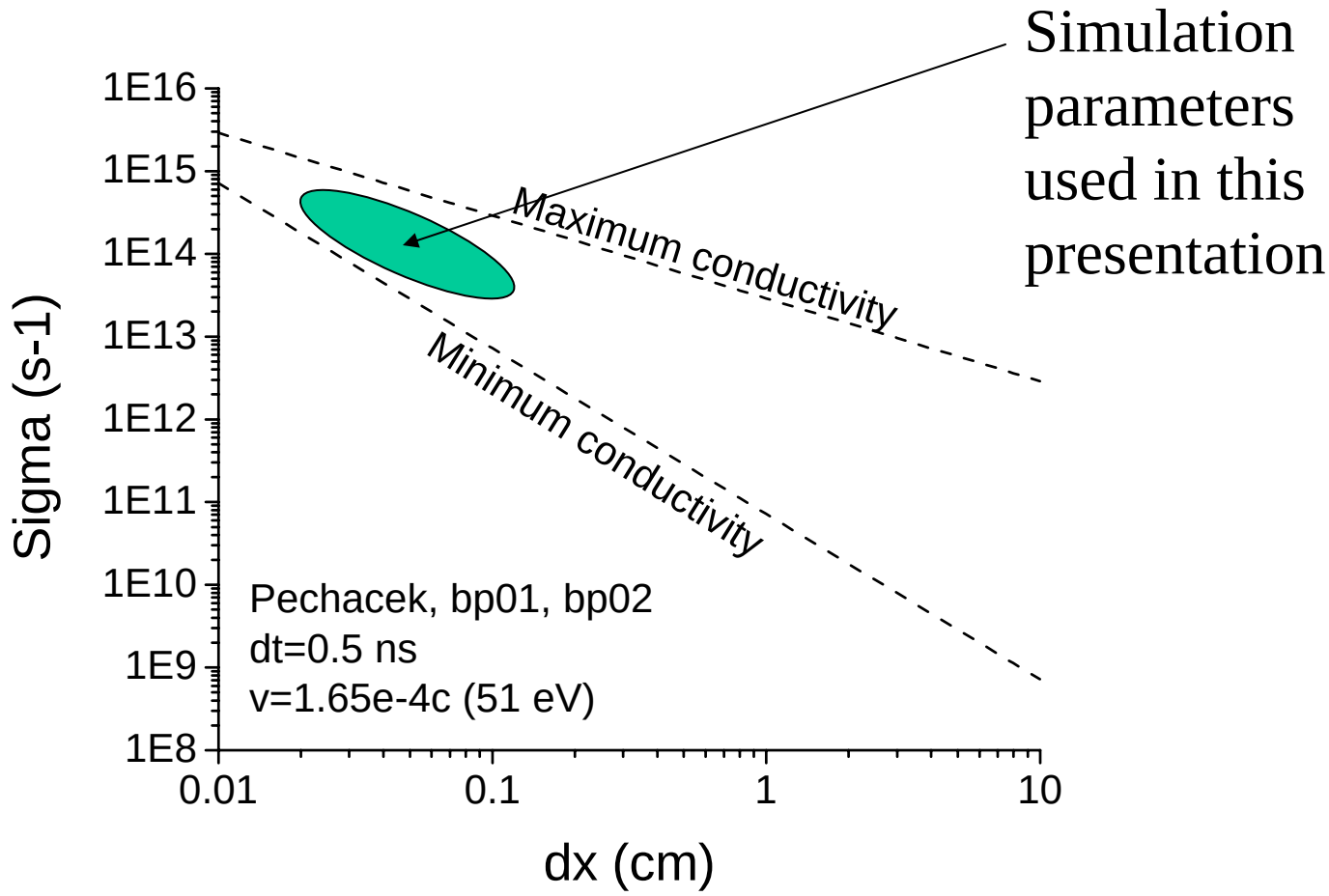


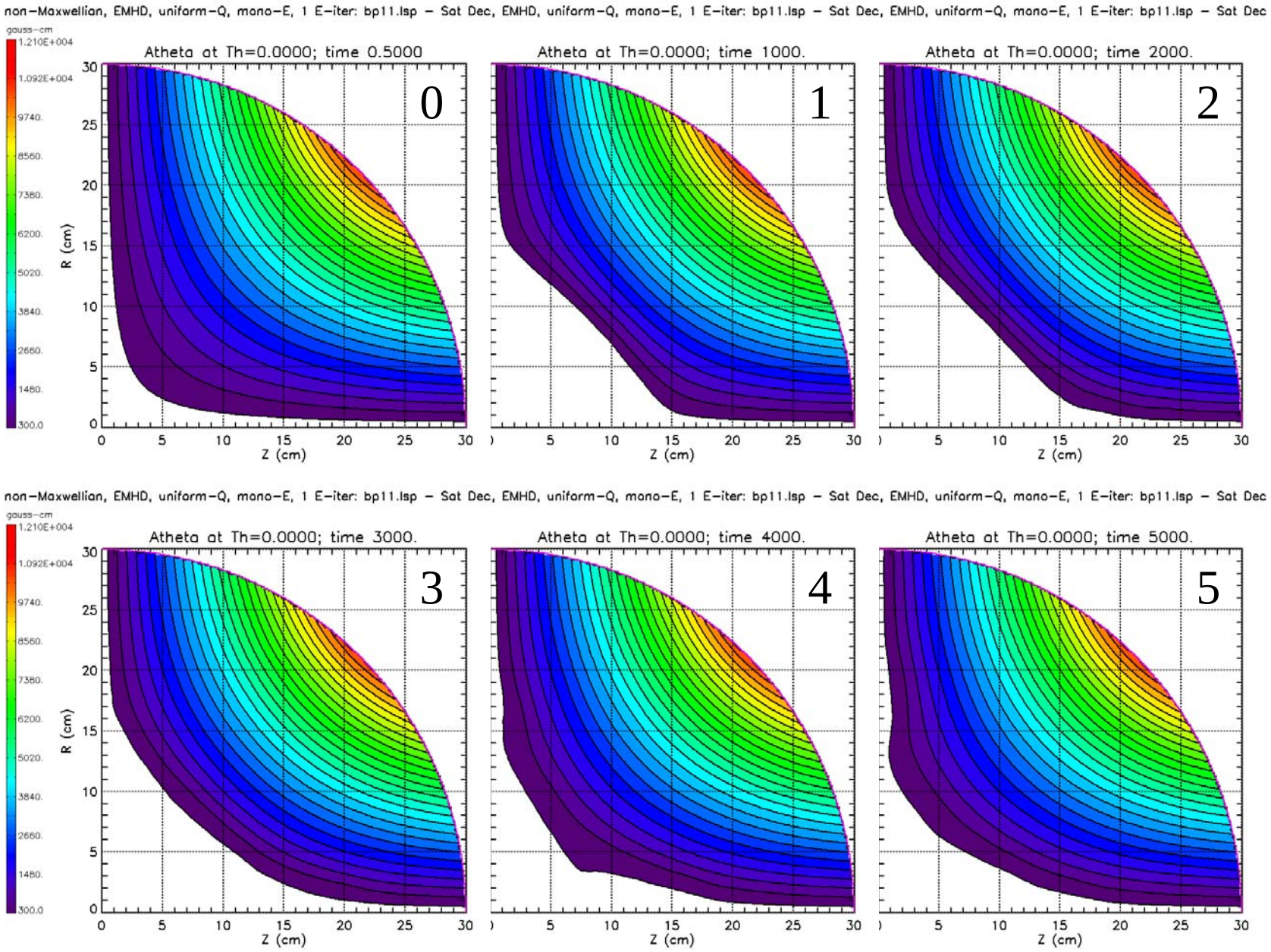
FIG. 1. Schematic diagram of the cusp experiment. The coil diameter is 70 cm. The scattered-light analyzing system and the incident-scattering laser pulse are actually in the same horizontal plane.

*R. E. Pechacek, *et al.*, Phys. Rev. Lett. **45**, 256 (1980).

The grid-Reynolds constraints suggests a reasonably wide parameter space for the Pechacek experiment

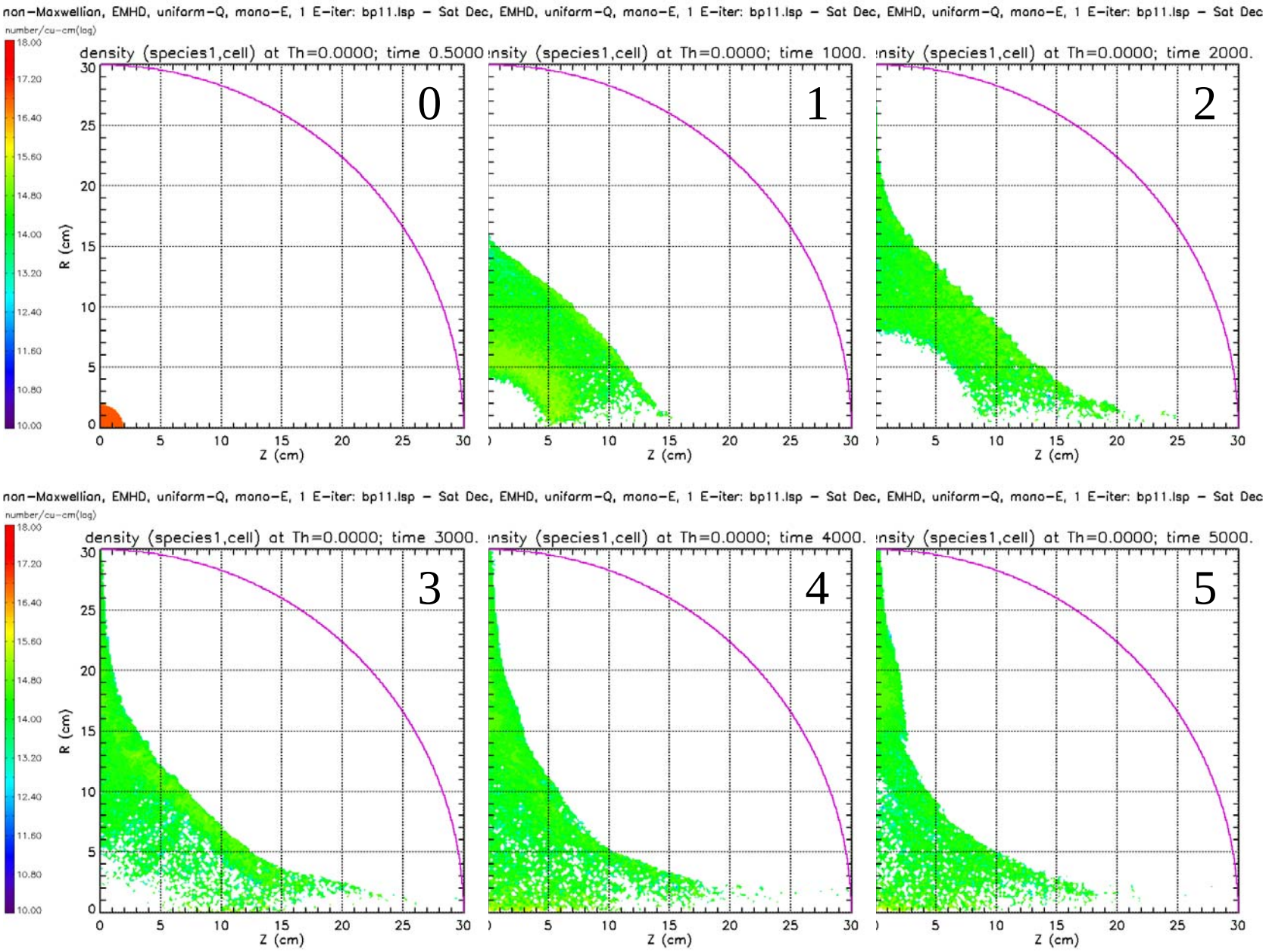


Magnetic field evolution ($|A_\theta|$):



Frames at *n* μs times

Ion Density Evolution:



Frames at n μs times

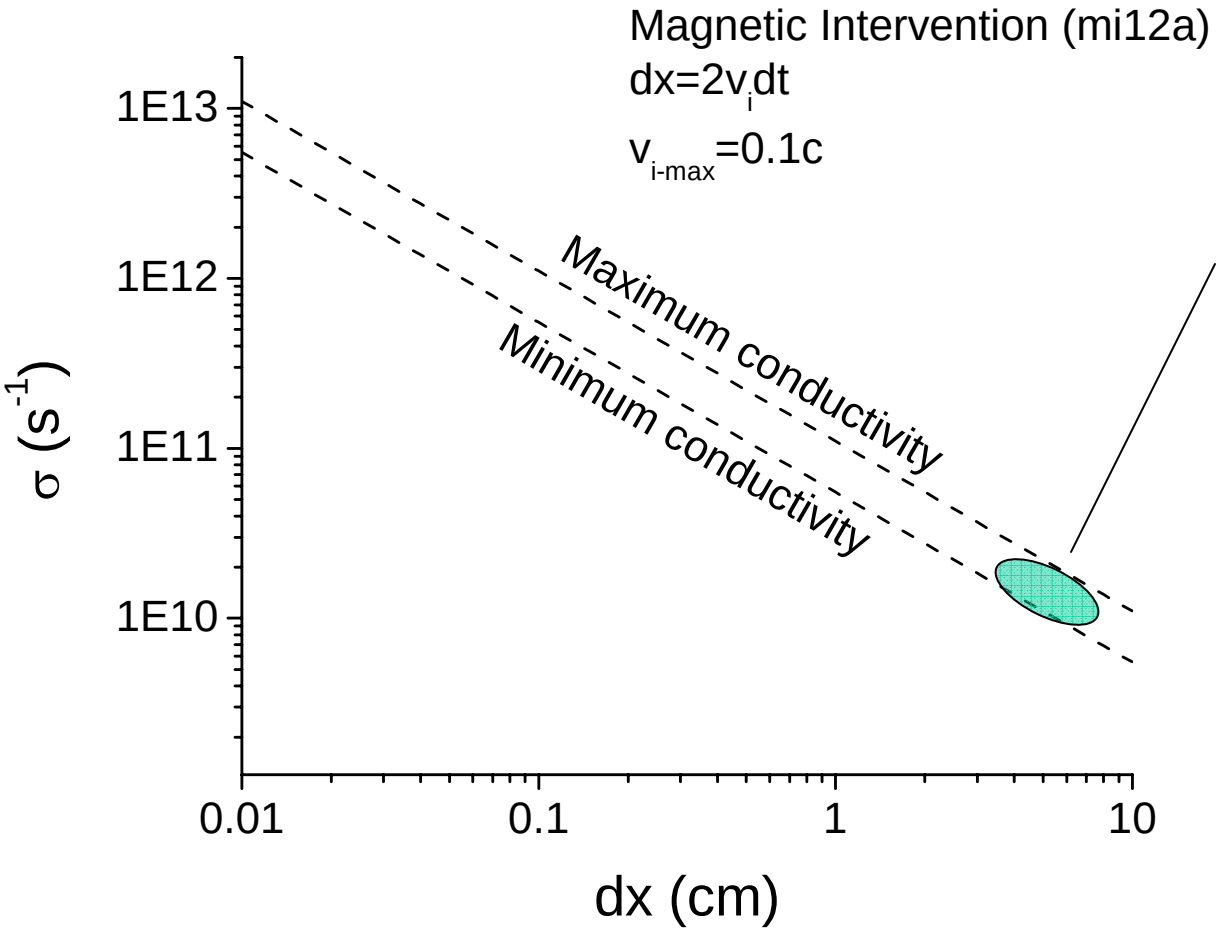
Status: Pechacek Experiment Modeling

- Present modeling is providing the best results to date, and detailed comparisons with the data are very compelling.
- Some problems resolved others remain with the EMHD solver (more work is required).
- Additional developments such as convergence testing and the use of canned, parallelized solvers (e.g. PETSC) are expected to make the algorithm faster.

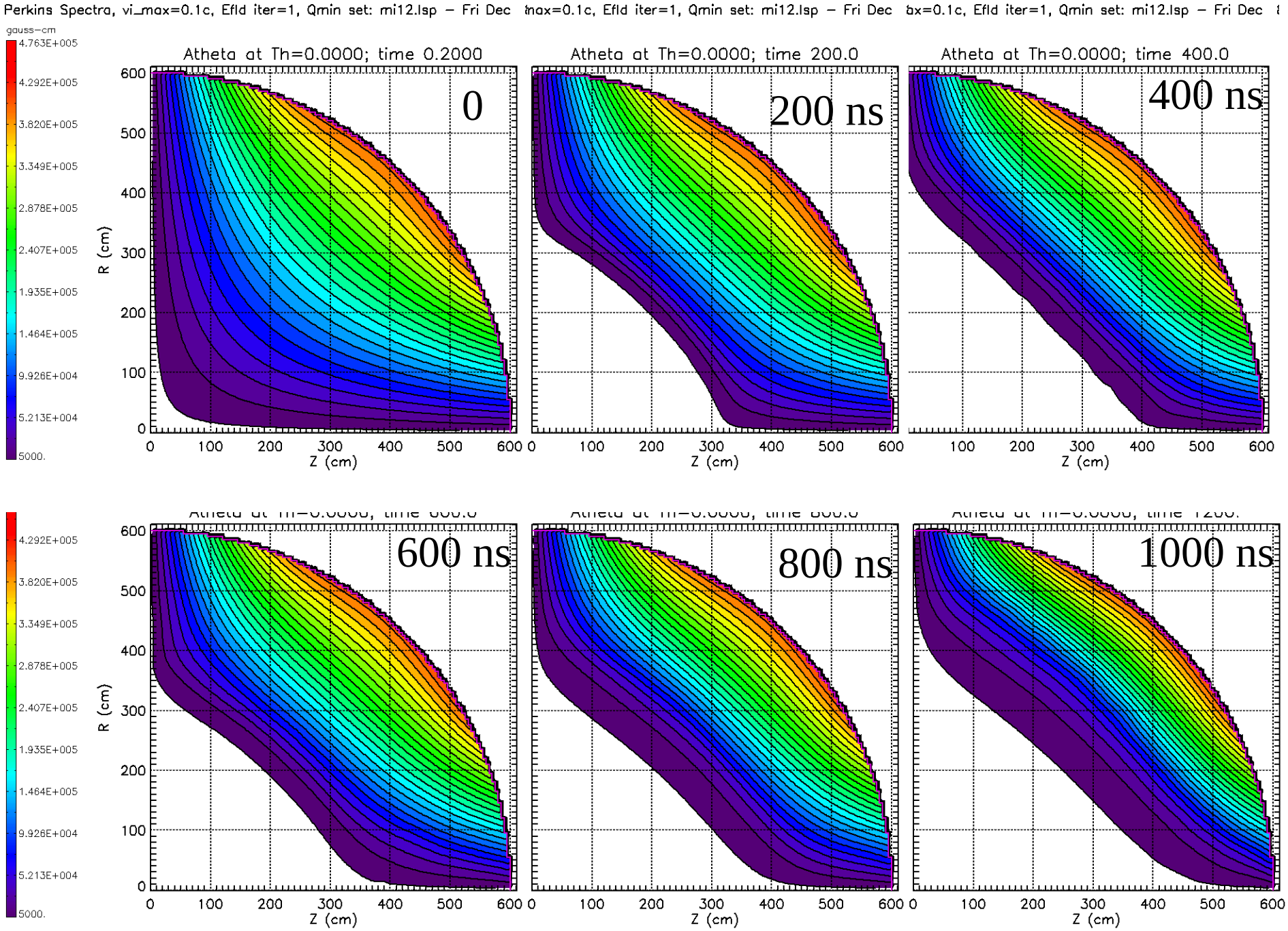
3. Chamber-Scale Magnetic Intervention Simulation:

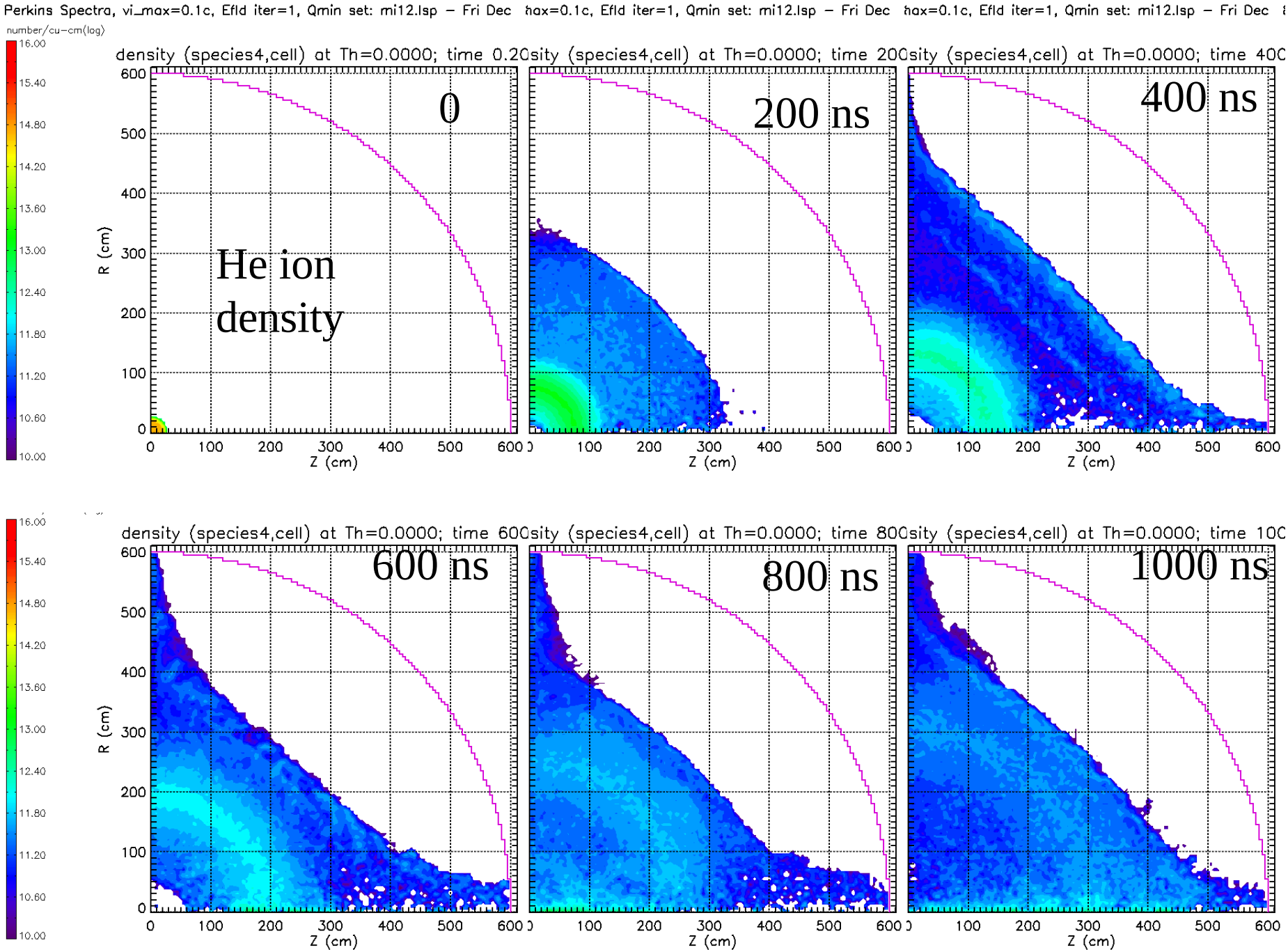
- A preliminary simulation result using the new solver and 5 species from the Perkins ion spectra is given (H, D, T, ^{4}He, ^{12}C).
- For computational expediency, ion distributions all truncated at v_i=0.1*c*.
- 4-coil magnetic field topology taken from previous “shell” model calculations of Robson and Genoni.

Computational constraints are <u>significant</u> for these ion speeds and scale lengths:

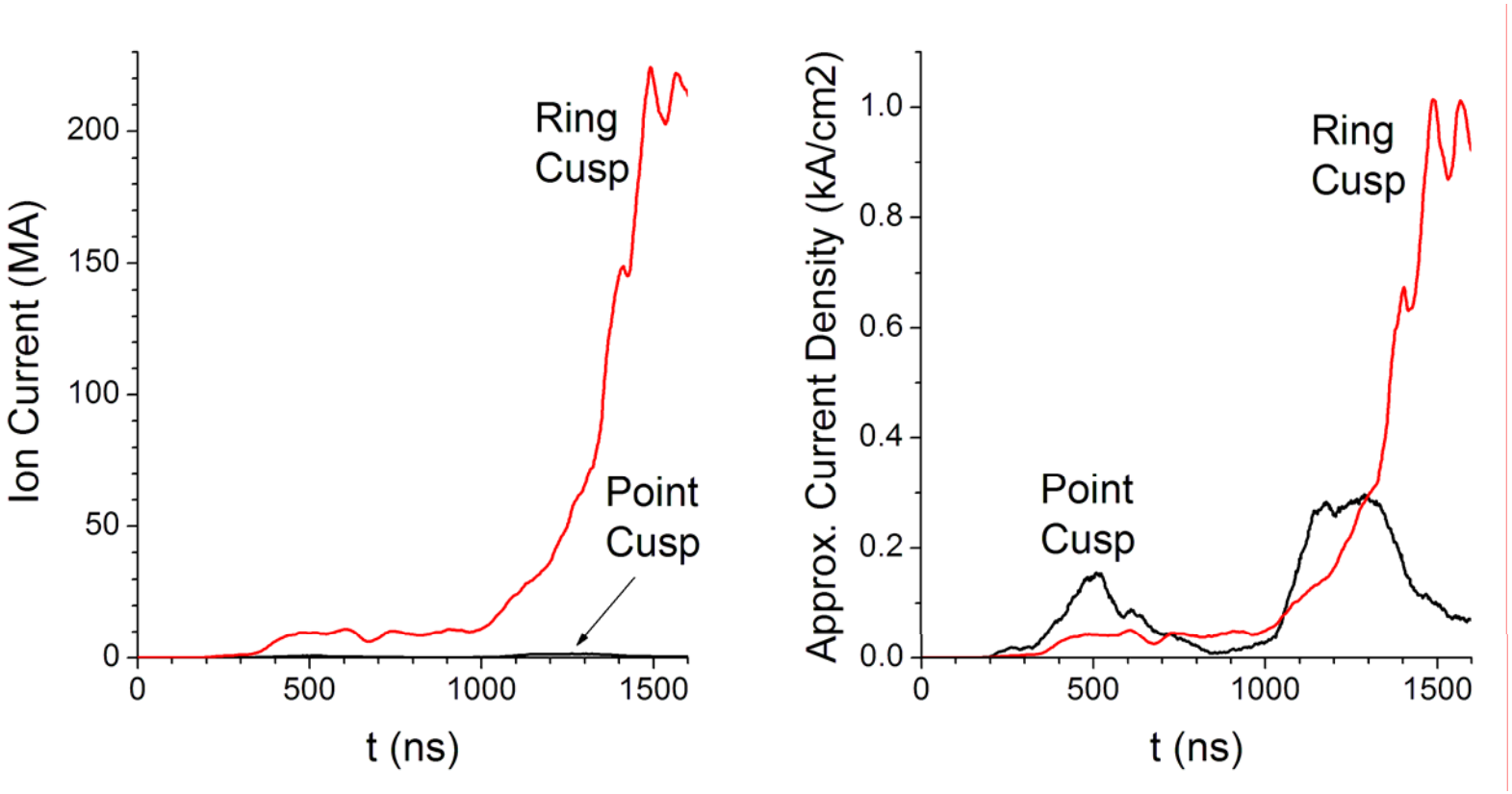


Parameter regime explored in this report.


Perkins Spectra, vi_max=0.1c, Efld iter=1, Qmin set: mi12.lsp – Fri Dec
gauss-cm
4.763E+005
4.292E+005
3.820E+005
3.349E+005
2.878E+005
2.407E+005
1.935E+005
1.464E+005
9.926E+004
5.213E+004
5000.
Atheta at Th=0.0000; time 0.2000
0
Atheta at Th=0.0000; time 200.0
200 ns
Atheta at Th=0.0000; time 400.0
400 ns
600 ns
800 ns
1000 ns
R (cm)
Z (cm)
0
100
200
300
400
500
600


Perkins Spectra, vi_max=0.1c, EfId iter=1, Qmin set: mi12.lsp – Fri Dec
number/cu-cm(log)
16.00
15.40
14.80
14.20
13.60
13.00
12.40
11.80
11.20
10.60
10.00
density (species4,cell) at Th=0.0000; time 0.2
density (species4,cell) at Th=0.0000; time 200
density (species4,cell) at Th=0.0000; time 400
0
200 ns
400 ns
He ion
density
R (cm)
Z (cm)
0
100
200
300
400
500
600
density (species4,cell) at Th=0.0000; time 600
density (species4,cell) at Th=0.0000; time 800
density (species4,cell) at Th=0.0000; time 100
600 ns
800 ns
1000 ns

Ion current densities at point and rings cusps are less than ~1 kA/cm (so far!)



Status: Magnetic Intervention Chamber Modeling

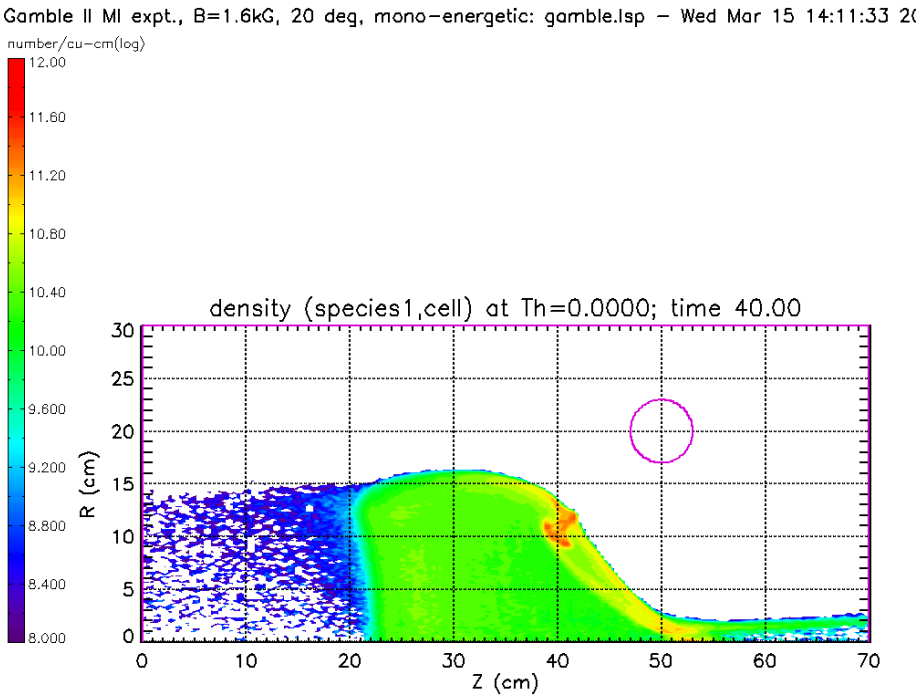
- A representative “shell” simulations are consistent with “shell” models of Robson and Genoni.
- Problems with the convergence of the solver (as seen in the Pechacek simulations) need to be resolved.
- Unlike the Pechacek experiment, the magnetic intervention parameter regime ion speeds and scale lengths may require *significant* computational resources using the EMHD algorithm. A parallel implementation of this algorithm is essential.

5: Next Steps…

- Continue to refine EMHD solver…
  - Non-uniform grids
  - Parallel implementation
- Complete analysis of Pechacek experiment and document
- Continue examination of magnetic intervention physics issues
  - Shock acceleration of ions at sheath…
  - Currently revisiting perfectly conducting “shell” model formulation for use on a fixed grid with arbitrary number of particles.

Extra slides

EMHD simulation used ~15,600 particles (protons)
Explicit PIC simulation used ~ 2,000,000 particles
(1,700,000 electrons, 300,000 protons)



Experimental Parameters

- Chamber wall radius is 30 cm (not shown)
- External field coils, 67 or 70 cm diam, 70 cm separation.
- |B| = 2.0 kG at ring cusp.
- 2×10^{19} "D_2" ions produced from cylindrical target of 1-mm diam., 1-mm length.
- Modeling assumes initial plasma is a thermal (51.1 eV), D^+ neutral plasma with initial radius of 2 cm.

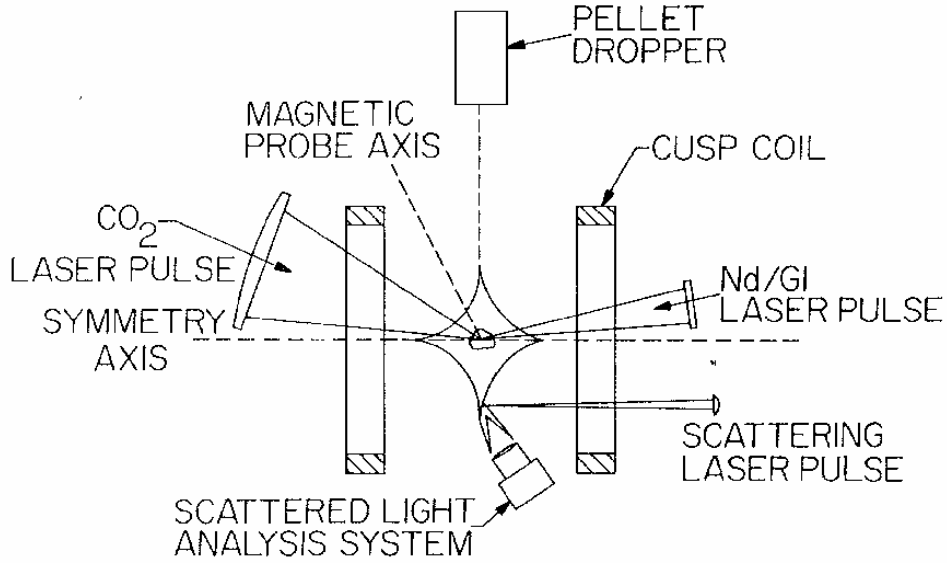


FIG. 1. Schematic diagram of the cusp experiment. The coil diameter is 70 cm. The scattered-light analyzing system and the incident-scattering laser pulse are actually in the same horizontal plane.

Plasma/Field boundary along 27 degree radial line from the cusp center (experimental result):

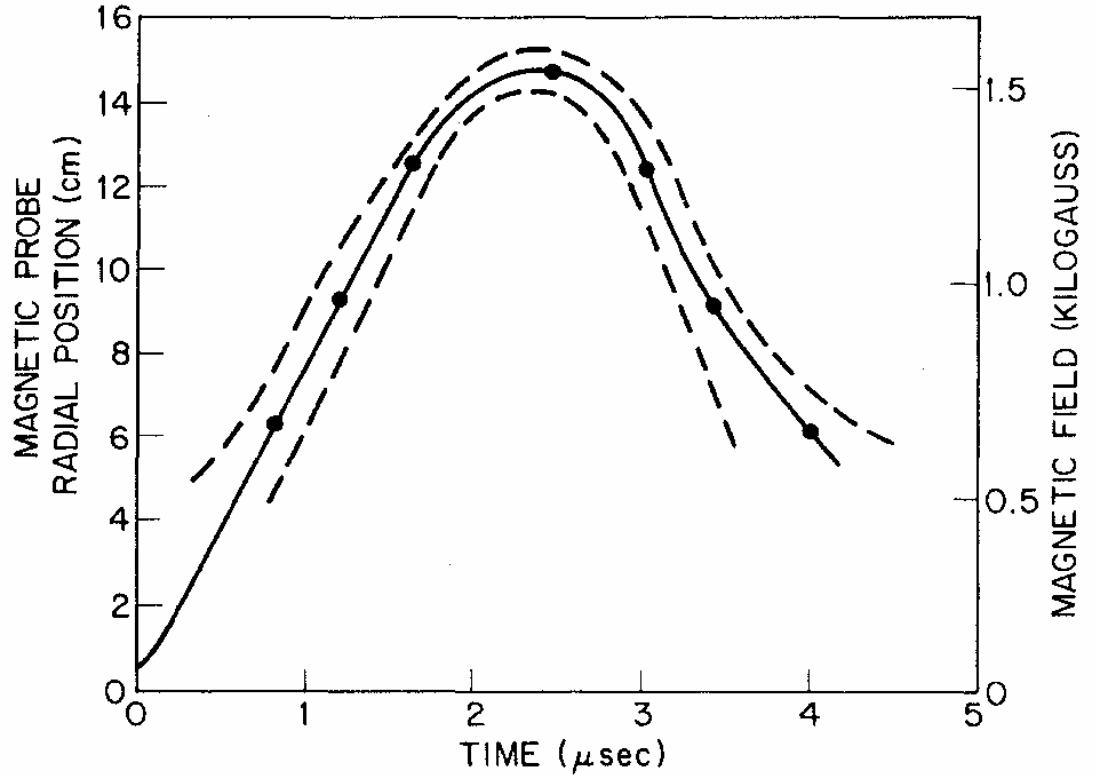


FIG. 3. Radial position of the plasma–magnetic-field interface vs time after the peak of the CO_2 laser pulse, derived from the time of arrival of the peak of the diamagnetic signal from a probe that is movable along a radial line inclined at 27° to the midplane. The dashed curves represent the half-peak points of the $\dot{B}$ signal.

At r=22 cm inside ring cusp, electron density was measured at 5 different times:

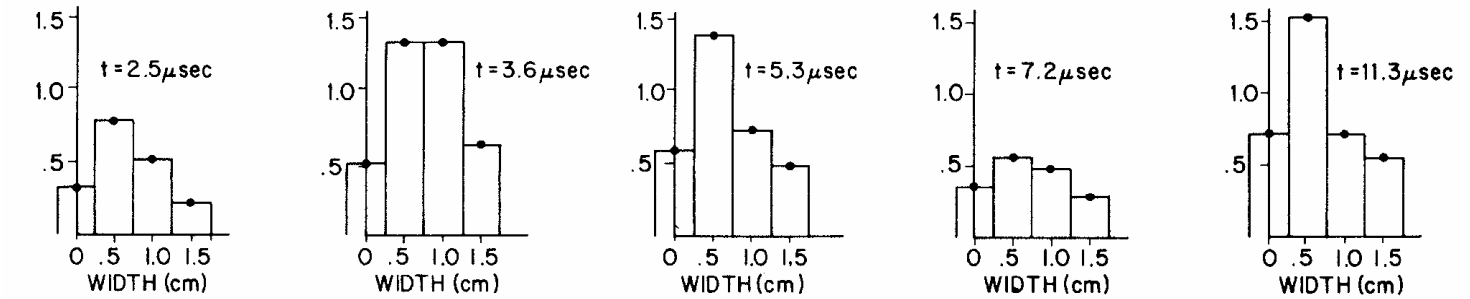
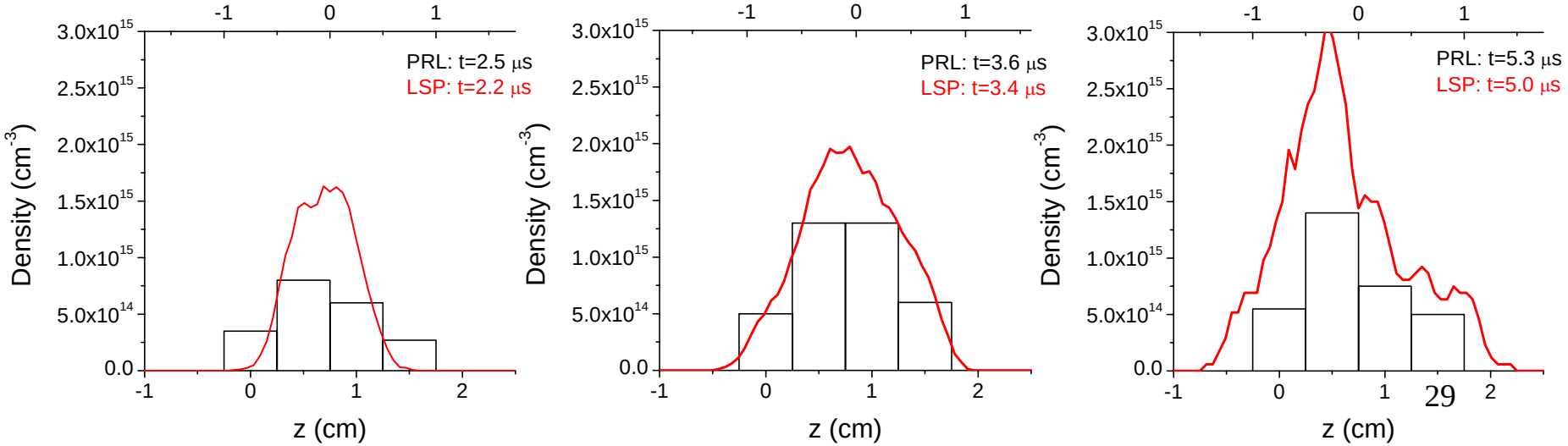


FIG. 4. Electron density as a function of position across the width of the ring cusp, at time t after the creation of the plasma. Each point is an average of two to four shots. The units of the vertical axes are 10^{15} cm^{-3}.

Simulations results in reasonable agreement with these measurements (at least for first 3 times):



Simulation dynamics are complex, but energy gain in all species (except protons?) is consistent with diffusion of plasma into sheath.