Dependence of ignition on the ion to electron energy confinement time ratio in FFHR

US-Japan Workshop on Fusion Power Plants and Related Advanced Technologies (UCSD, 2012.3.8-9)

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1. Introduction

- **So far one fluid equation (T_i=T_e) has been used in ignition access in FFHR.**

- **Effect of two fluid equations (T_i, T_e) (the separate ion and electron power balance equations) on ignition access in FFHR have been studied in the high temperature and low density and the low temperature and high-density regimes.**

- **In the thermally stable regime, ignition is more difficult than the case using one fluid equation when ion confinement time τ_{Ei} is close to the electron confinement time τ_{Ee}.**

- **In the thermally unstable regime, ignition is not sensitive to two fluid equations.**

2. Formalism for two fluids 0-D equations

(1) Particle balance equations

- $$\frac{dn_D(0)}{dt} = (1+\alpha_n)S_D(t) - (1+\alpha_n)n_D(0)n_T(0)\overline{<\sigma V>_{DT}(x)} - \frac{n_D(0)}{\tau_D^*} \quad \textbf{(2-1)}$$

- $$\frac{dn_T(0)}{dt} = (1+\alpha_n)S_T(t) - (1+\alpha_n)n_D(0)n_T(0)\overline{<\sigma V>_{DT}(x)} - \frac{n_T(0)}{\tau_T^*} \quad \textbf{(2-2)}$$

- $$\frac{dn_\alpha(0)}{dt} = (1+\alpha_n)n_D(0)n_T(0)\overline{<\sigma V>_{DT}(x)} - \frac{n_\alpha(0)}{\tau_\alpha^*} \quad \textbf{(2-3)}$$

(2) Power balance equations:

The ion power balance:

- $$\frac{dW_i}{dt} = \bar{P}_\alpha F_{\alpha ion} + P_{EXT}F_{Pi}/V_o - \bar{P}_{ie} - \bar{P}_{Li} \quad \textbf{(2-4)}$$

The electron power balance:

- $$\frac{dW_e}{dt} = \bar{P}_\alpha(1-F_{\alpha ion}) + P_{EXT}(1-F_{Pi})/V_o + \bar{P}_{ie} - \bar{P}_b - \bar{P}_s - \bar{P}_{Le} \quad \textbf{(2-5)}$$

where P_{ie} is the power transfer from the ion to electron:

$$\bar{P}_{ie}\left[W/m^3\right] = \frac{2.4\times10^5\left(n_e(0)[m^{-3}]/10^{20}\right)^2}{1+2\alpha_n-0.5\alpha_T}\left(\frac{f_D}{A_D}+\frac{4f_\alpha}{A_\alpha}+\frac{f_T}{A_T}\right)\ln\Lambda_j\frac{1}{T_e(0)[keV]^{0.5}}\left(\frac{T_i(0)}{T_e(0)}-1\right) \quad \textbf{(2-6)}$$

(A_D, A_α and A_T are the mass number of deuteron, Helium-4 and triton, $\ln\Lambda$=20).

The ion and the electron conduction loss :

$$\bar{P}_{Li} = \frac{\frac{3}{2}k\left(n_D + n_T + n_\alpha\right)T_i}{\tau_{Ei}} \quad , \quad \bar{P}_{Le} = \frac{\frac{3}{2}k\left(n_e\right)T_e}{\tau_{Ee}} \qquad \textbf{(2-7)}$$

The sum of these conduction losses is the total plasma conduction loss

$$\bar{P}_{Li} + \bar{P}_{Le} = \bar{P}_L \qquad \textbf{(2-8)}$$

Using the ratio of the ion to electron confinement time loss $\gamma_{ci} = \tau_{\mathrm{Ei}} / \tau_{\mathrm{Ei}}$

$$\frac{1.5e\left(f_D + f_T + f_\alpha\right)n(0)T_i(0)}{\gamma_{ci}\tau_{Ee}\left(1+\alpha_n+\alpha_T\right)} + \frac{1.5en(0)T_e(0)}{\tau_{Ee}\left(1+\alpha_n+\alpha_T\right)} = \bar{P}_L \qquad \textbf{(2-8')}$$

The electron confinement time τ_{Ee} is obtained, and then the ion confinement time τ_{Ei} can be calculated.

The ion conduction loss and the electron conduction loss :

$$\bar{P}_{Li}[M/m^3] = \frac{\left(f_D + f_T + f_\alpha\right)}{\gamma_{ci}\left\{\frac{\left(f_D + f_T + f_\alpha\right)}{\gamma_{ci}} + \frac{T_e(0)}{T_i(0)}\right\}}\bar{P}_L \quad , \quad \bar{P}_{Le}[W/m^3] = \left[1 - \frac{\left(f_D + f_T + f_\alpha\right)}{\gamma_{ci}\left\{\frac{\left(f_D + f_T + f_\alpha\right)}{\gamma_{ci}} + \frac{T_e(0)}{T_i(0)}\right\}}\right]\bar{P}_L \qquad \textbf{(2-7')}$$

The ion temperature and electron temperature are calculated by, respectively,

- $$\frac{dT_i(0)}{dt} = \frac{(1+\alpha_n+\alpha_T)}{1.5\times1.6\times10^{-19}\left(f_D(0)+f_\alpha(0)+f_T(0)\right)n_e(0)}\left[\bar{P}_\alpha F_{\alpha ion} + P_{EXT}F_{NBIion}/V_o - \bar{P}_{ie} - \bar{P}_{Li}\right] - \frac{T_i(0)}{\left(f_D(0)+f_\alpha(0)+f_T(0)\right)}\frac{1}{n_e(0)}\left[\frac{dn_D(0)}{dt}+\frac{dn_\alpha(0)}{dt}+\frac{dn_T(0)}{dt}\right] \quad \textbf{(2-4')}$$

- $$\frac{dT_e(0)}{dt} = \frac{(1+\alpha_n+\alpha_T)}{1.5\times1.6\times10^{-19}n_e(0)}\left[\bar{P}_\alpha\left(1-F_{\alpha ion}\right)+P_{EXT}\left(1-F_{NBIion}\right)/V_o + \bar{P}_{ie} - \bar{P}_s - \bar{P}_b - \bar{P}_{Le}\right] - \frac{T_e(0)}{\left(1-(1+\alpha_n)Zf_{imp}\right)}\frac{1}{n_e(0)}\left[\frac{dn_D(0)}{dt}+2\frac{dn_\alpha(0)}{dt}+\frac{dn_T(0)}{dt}\right] \quad \textbf{(2-5')}$$

- **ISS95 scaling**

$$\begin{cases}\tau_E[s] = \gamma_{ISS}\,\tau_{ISS95}[s] = \gamma_{LHD}\times1.6\;\tau_{ISS95}[s] \\ \tau_{ISS95}[s] = 0.079\,\iota_{2/3}^{0.4}\,\bar{n}_{19}^{\;0.51}[10^{19}\,m^{-3}]B_o^{0.83}[T]\bar{a}^{2.21}[m]R^{0.65}[m]/P_{HT}^{0.59}[MW]\end{cases}$$

- **External heating power (Thermally stable regime)**

$$P_{EXT}[W] = \left\{\gamma_{pr}\frac{\left[\gamma_{DLM}n(0)[m^{-3}]\right]}{\gamma_{SUDO}\,0.25\times10^{20}}\right\}^2\frac{\bar{a}^2R[m]}{B_o[T]}\times10^6 - \left(P_\alpha - P_B - P_S\right)$$

- **Machine parameters:**

R=15.7m, a=2.5m, P_f=3 GW, η_α=0.98, B=4.5T, NBI (1.5MeV) and ECRH (126 GHz, n_c=1.97x10^{20} m^{-3})

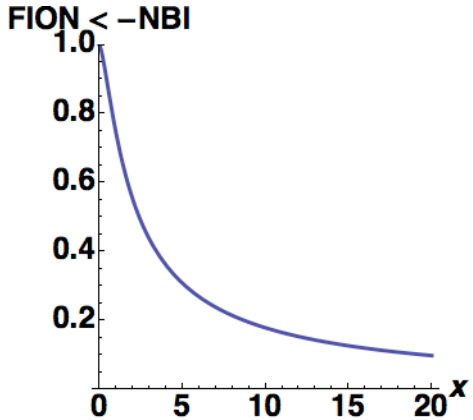
Thermally stable regime : α_n=0.5, α_T=1.0, γ_{ISS}=1.92

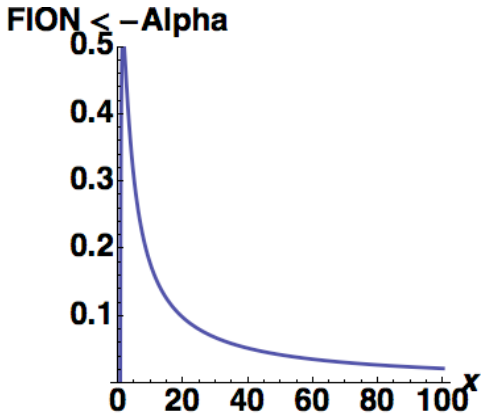
Thermally unstable regime : α_n=3.0, α_T=1.0, γ_{ISS}=1.43

(3) Heating power fraction to ion:

- **ECRH: $F_{pi}=0$, $F_{pe}=(1-F_{pi})=1$**
- **NBI of 1.5 MeV:** $x_o = \frac{E_{NBI0}}{E_{crit}} = \frac{1500(keV)}{16\cdot T_e(0)/(1+\alpha_T)(keV)}$ **（Wesson, “Tokamaks”）**

$$F_{NBIion} = \frac{E_{crit}}{E_{NBI0}} \int^{E_{NBI0}/E_{crit}} \frac{d\left(E_{NBI}/E_{crit}\right)}{1+\left(E_{NBI}/E_{crit}\right)^{1.5}} = \frac{1}{x_o}\int_0^{1/x_o} \frac{dx}{1+x^{1.5}} = \frac{1}{x_o}\left[\frac{1}{3}\log\frac{1-\sqrt{x_o}+x_o}{\left(1+\sqrt{x_o}\right)^2} + \frac{2}{\sqrt{3}}\left(\tan^{-1}\frac{2\sqrt{x_o}-1}{\sqrt{3}} + \frac{\pi}{6}\right)\right]$$





(Typically: stable: T~16 keV, x_o=11.7, F_{NBIion}~0.16)
(Typically: unstable: T~10 keV, x_o=20.0, F_{NBIion}~0.10)

(Typically: stable: T~16 keV, $x_{\alpha o}$=27.3, $F_{\alpha ion}$~0.07)
(Typically: unstable: T~7 keV, $x_{\alpha o}$=62.5, $F_{\alpha ion}$~0.03)

- **Alpha heating of 3.5 MeV:**

(ref: C. Neumeyer, M. Peng, P. Rutherford, "Spherical Torus Design Point Studies" June, 2003)

$$x_{\alpha o} = \frac{E_{\alpha 0}}{E_{crit}} = \frac{3500(keV)}{16\cdot T_e(0)/(1+\alpha_T)(keV)}$$

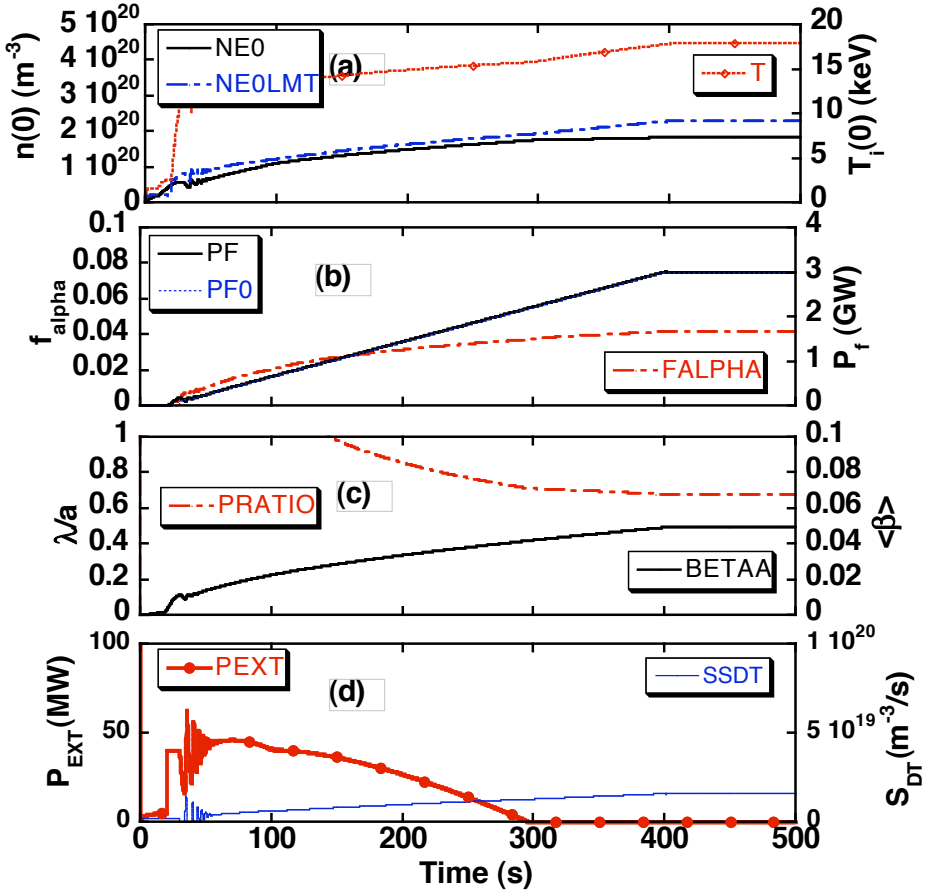
$$F_{\alpha ion} = 1 - \frac{E_{crit}}{E_{\alpha 0}} \int_0^{E_{\alpha 0}/E_{crit}} \frac{d\left(E_\alpha/E_{crit}\right)}{1+\left(E_\alpha/E_{crit}\right)^{1.5}} = 1 - \frac{1}{x_{\alpha o}}\int_0^{1/x_{\alpha o}} \frac{dx_\alpha}{1+x_\alpha^{1.5}} = 1 - \left(1 - \frac{2.4}{x_{\alpha o}} + \frac{2.0}{x_{\alpha o}^{1.5}}\right)$$

3. Ignition access to the thermally stable regime

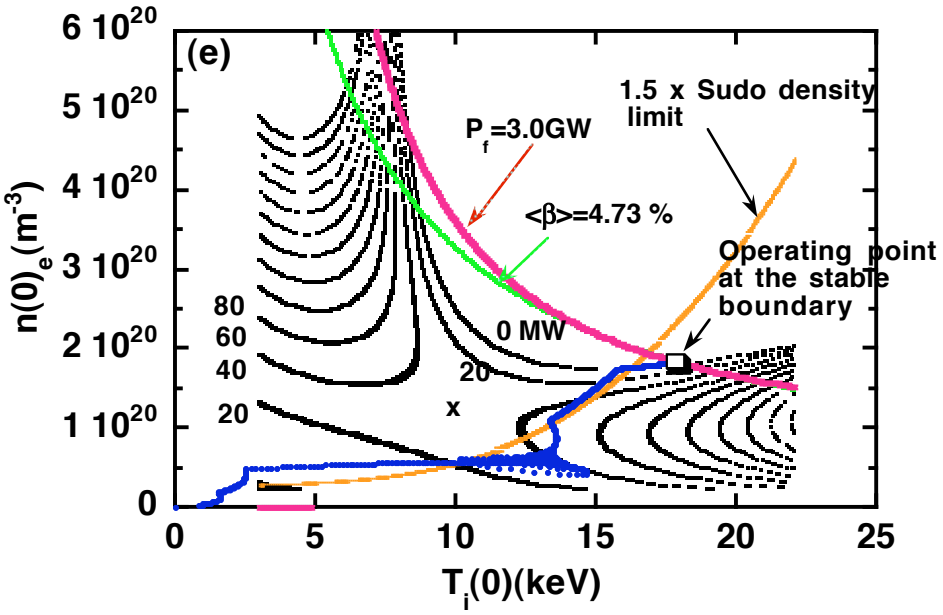
3.1. One fluid equation

Heating power 40 MW, γ_{ISS}=1.92 (γ_{LHD}= 1.2), B=4.5T, n(0)~1.83x10^{20} m^{-3} < n_c, T(0)~17.8 keV, Γ_n~ 1.5 MW/m^2, <β>~4.9 %, Beam penetration length λ/a~ 0.7（E_{NBI}=1.5 MeV）

Heating power is zero at 300 s.



Maximum heating power is ~45 MW.



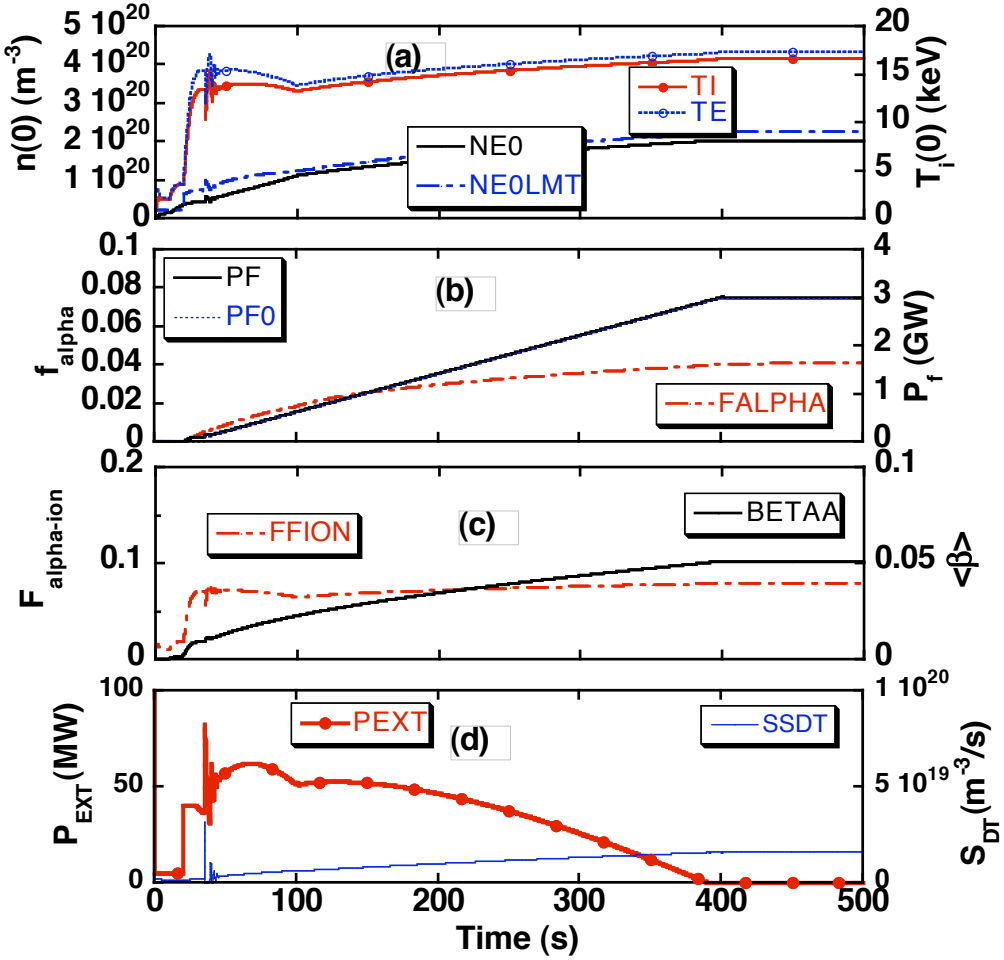
3.2. Two fluid equations: Ignition access by ECRH alone

(100 % electron heating by ECRH, 92 % electron heating by alpha heating)

[3.2.1] $\gamma_{ci} = \tau_{Ei}/\tau_{Ee} = 2$

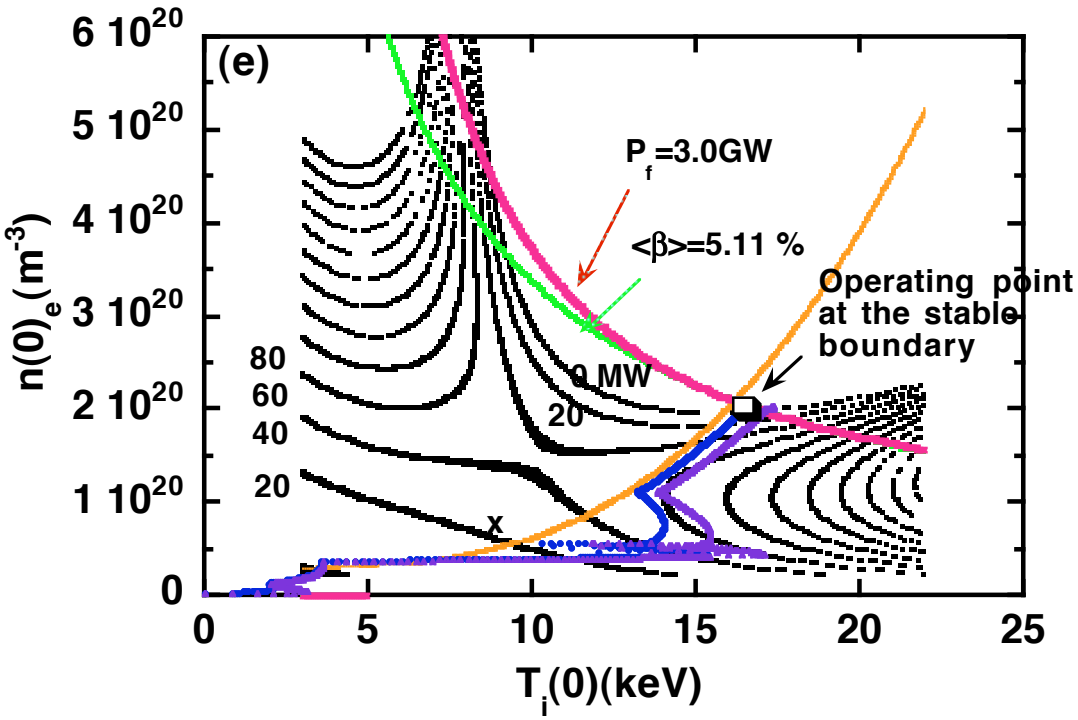
$F_{ECRH\text{-}ion}$= 0, $F_{alpha\text{-}ion}$ =0.08 → T_{i0}= 16.65 keV, T_{e0}= 17.35 keV, T_i/T_e=0.96,

n(0)~1.83x10^{20} m^{-3} < n_c, P_{Li}=142 MW, P_{Le}=340 M, P_{ie}= - 95 MW



- **ECRH works, but the heating power is increased than one fluid case and ignition is delayed.**
- **Oscillation of the heating power is removed by adjusting initial heating power, initial fueling rate, and derivative time.**
- **Maximum heating power is 60 MW.**
- **Temperature difference is large in the low-density phase less than 1x10^{-20} m^{-3}.**
- **Steady state temperature ratio is determined by the confinement time ratio and alpha heating fraction.**

POPCON on (n-T_i) plane



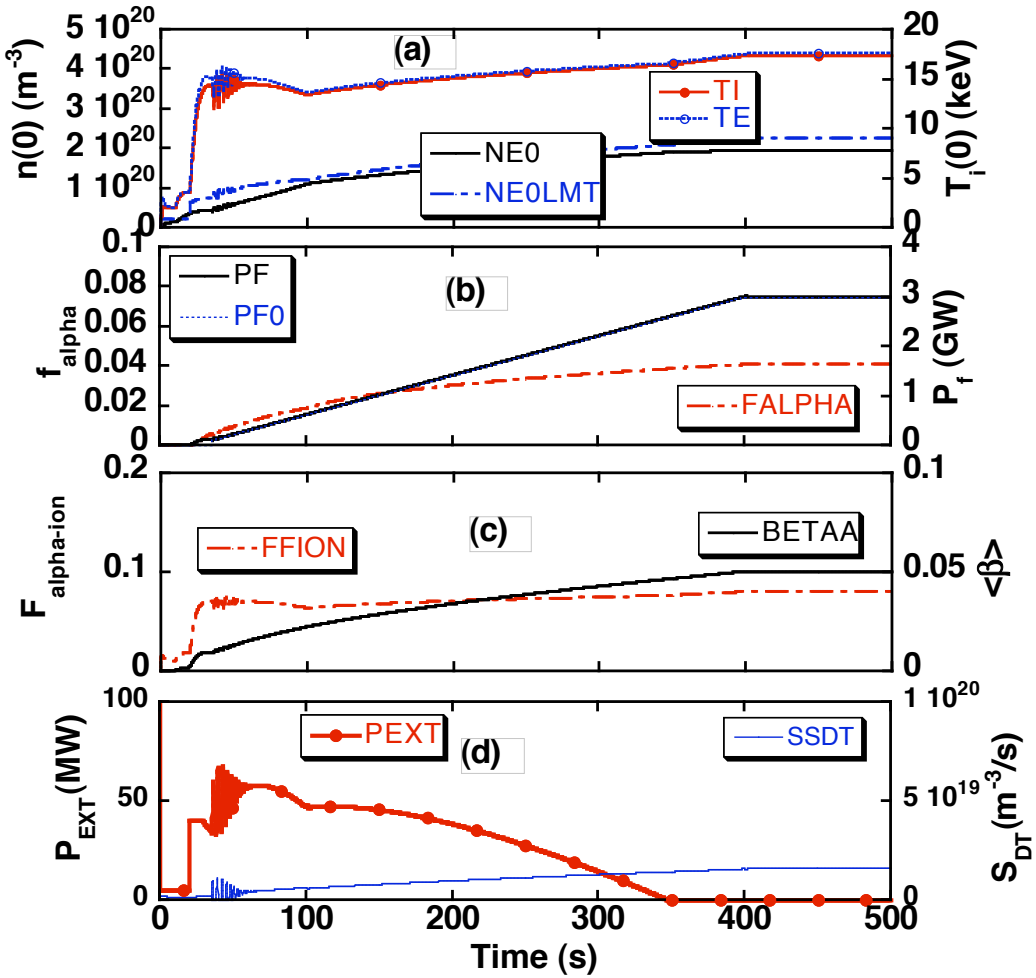
Blue: Ion temperature

Purple: Electron temperature

[3.2.2] $\gamma_{ci} = \tau_{Ei}/\tau_{Ee} = 5$

$F_{ECRH\text{-}ion}$= 0, $F_{alpha\text{-}ion}$ =0.08, → T_{i0}= 17.38 keV, T_{e0}= 17.58 keV, T_i/T_e=0.9886,

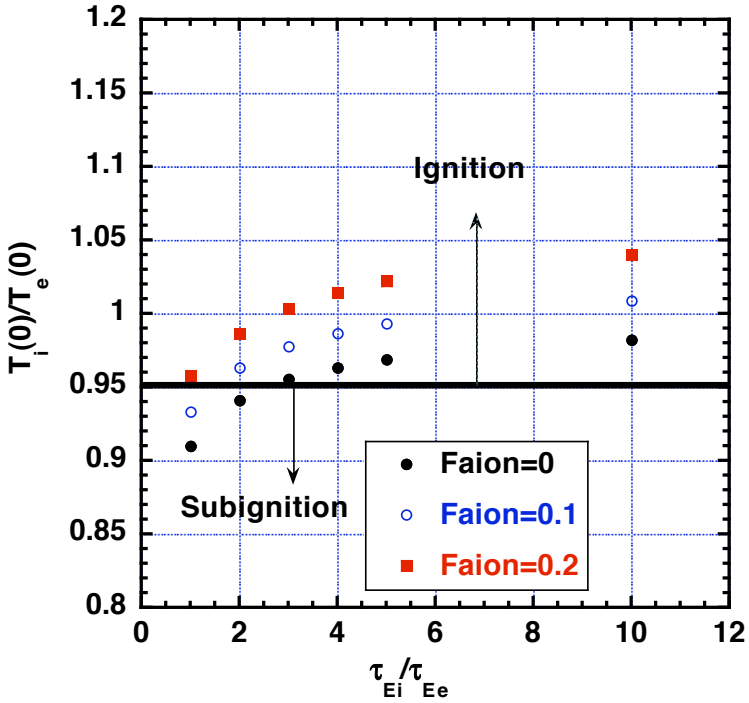
n(0)~1.83x10^{20} m^{-3} < n_c P_{Li}=71 MW, P_{Le}=417 M, P_{ie}= - 24 MW

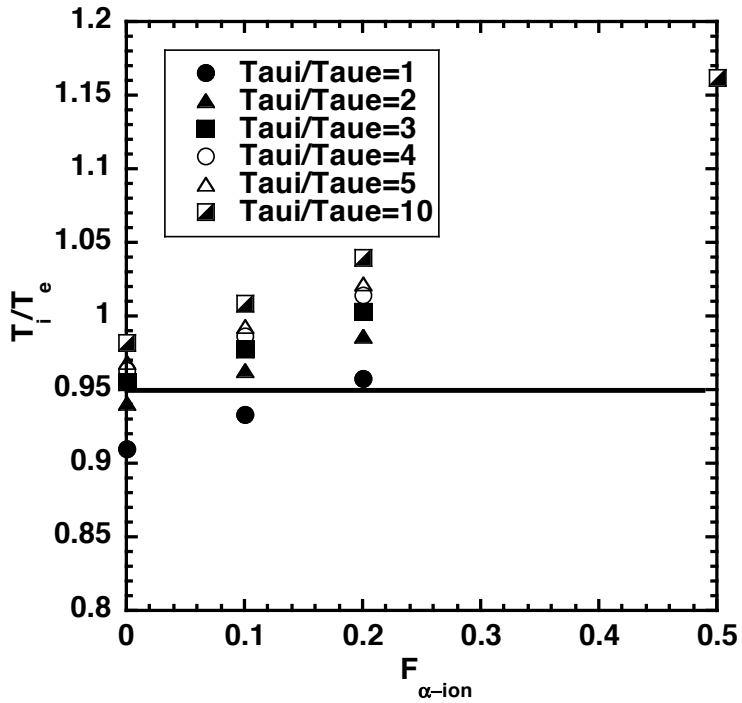


● Ion temperature is getting close to the electron temperature due to better ion confinement.

[3.2.3] Dependence of the T_i/T_e on confinement time ratio in the stable regime

ECRH ($F_{ECRH.ion}$=0), $F_{alpha\text{-}ion}$ = 0, 0.1, 0.2





- [left] It is difficult to achieve ignition by hot electron mode.

- Improvement of the ion confinement time does not much increase the ion temperature .
  (Ion temperature increment saturates with respect to the ion confinement time.
  On the other hand, the ion temperature increases linearly with the ion heating fraction.)

- For alpha heating fraction to ions $F_{alpha\text{-}ion}$~0.08, $\tau_{Ei}/\tau_{Ee} > 2$ is necessary for ignition.

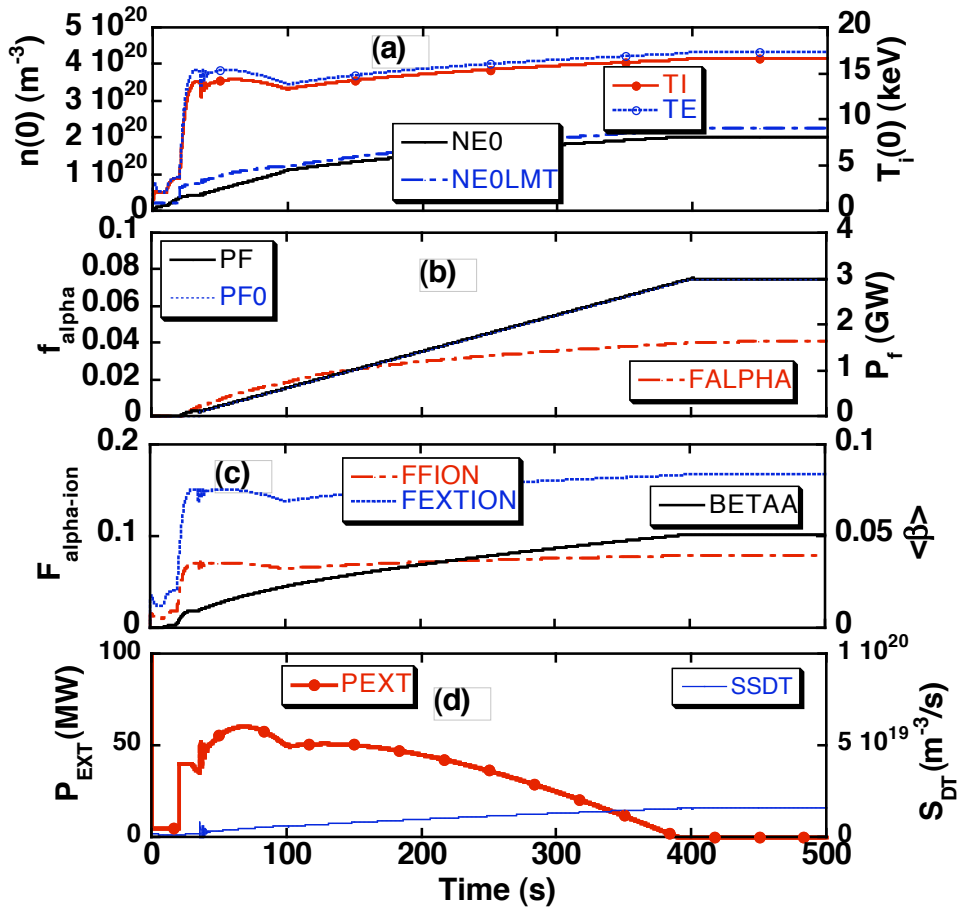
3.3. Two fluid equations: Ignition access by NBI

(80 % electron heating by NBI, 92% electron heating by alpha heating)

[3.3.1] $\gamma_{ci} = \tau_{Ei}/\tau_{Ee} = 2$

$F_{NBI\text{-}ion}$=0.2, $F_{alpha\text{-}ion}$ =0.08 → Steady state: T_i/T_e=0.96

The same as ECRH case except for the phase of n<1x10^{20} m^{-3}.

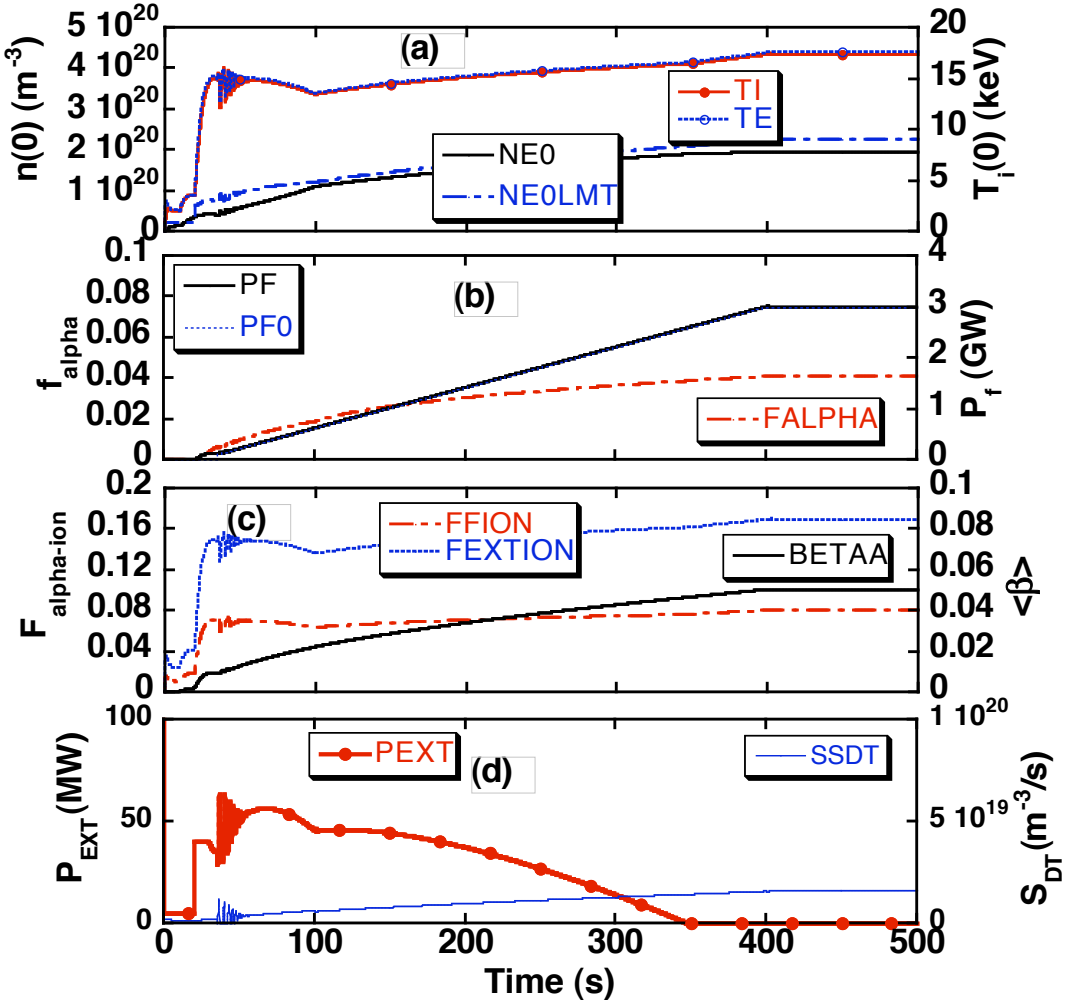


- **Confinement time ratio determines the ignition characteristics.**
- **Almost the same in ECRH and NBI heating except for the phase of n<1x10^{20} m^{-3}.**

[3.3.2] $\gamma_{ci} = \tau_{Ei}/\tau_{Ee} = 5$

$F_{NBI\text{-}ion}$=0.2, $F_{alpha\text{-}ion}$ =0.08, → Steady state: The same as ECRH T_i/T_e=0.99

The same as ECRH case except for the phase of n<1x10^{20} m^{-3}.



●Similar to the one fluid equation.

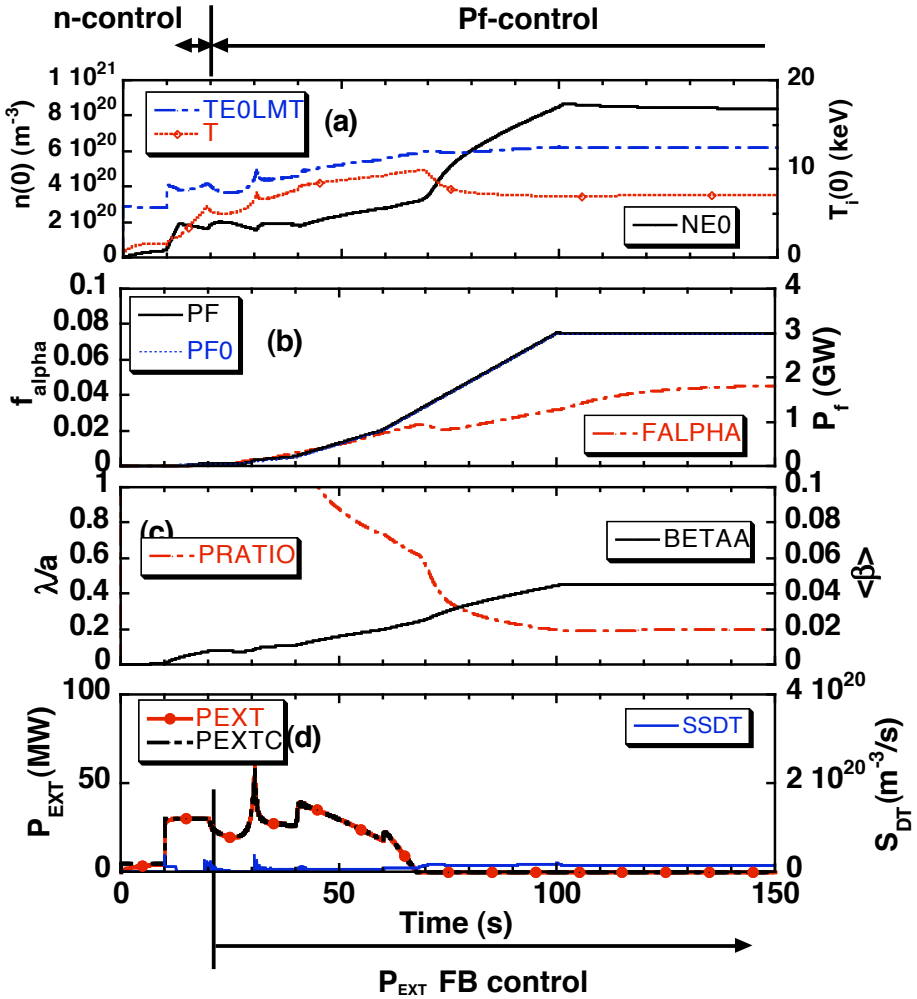
4. Ignition access to the thermally unstable regime

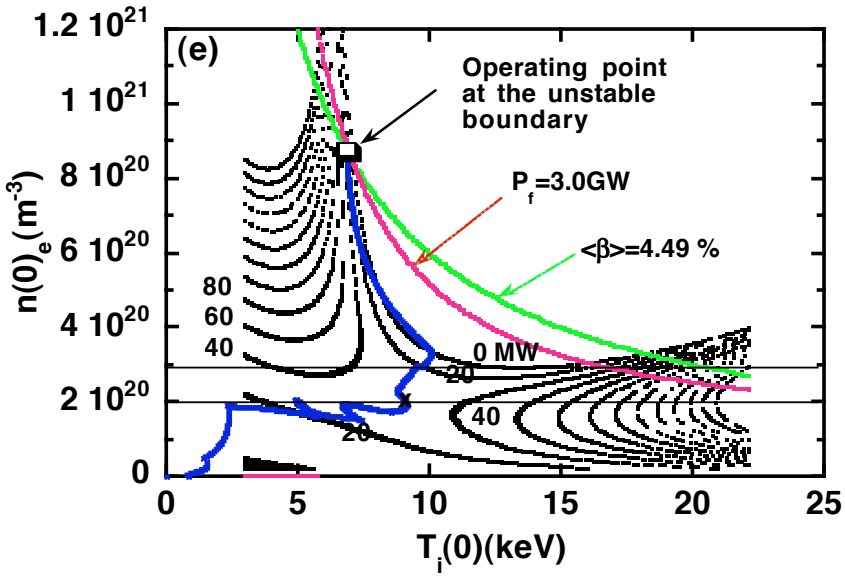
4.1. One fluid equations

[4.1.1] Feedback controlled ignition access by newly developed algorithm

γ_{ISS}=1.43, B=4.5, α_n=3, α_T=1.0, E=1.5MeV TanNBI, n(0)=8.40x0^{20} m^{-3}, T(0)=7.1 keV, f_α=4.5 %, <β>=4.5 %,

P_{div}=12.2 MW/m^2 (θ=90°, Δ=0.1 m) T_c= 7.7 keV, a_{TC}=0.2

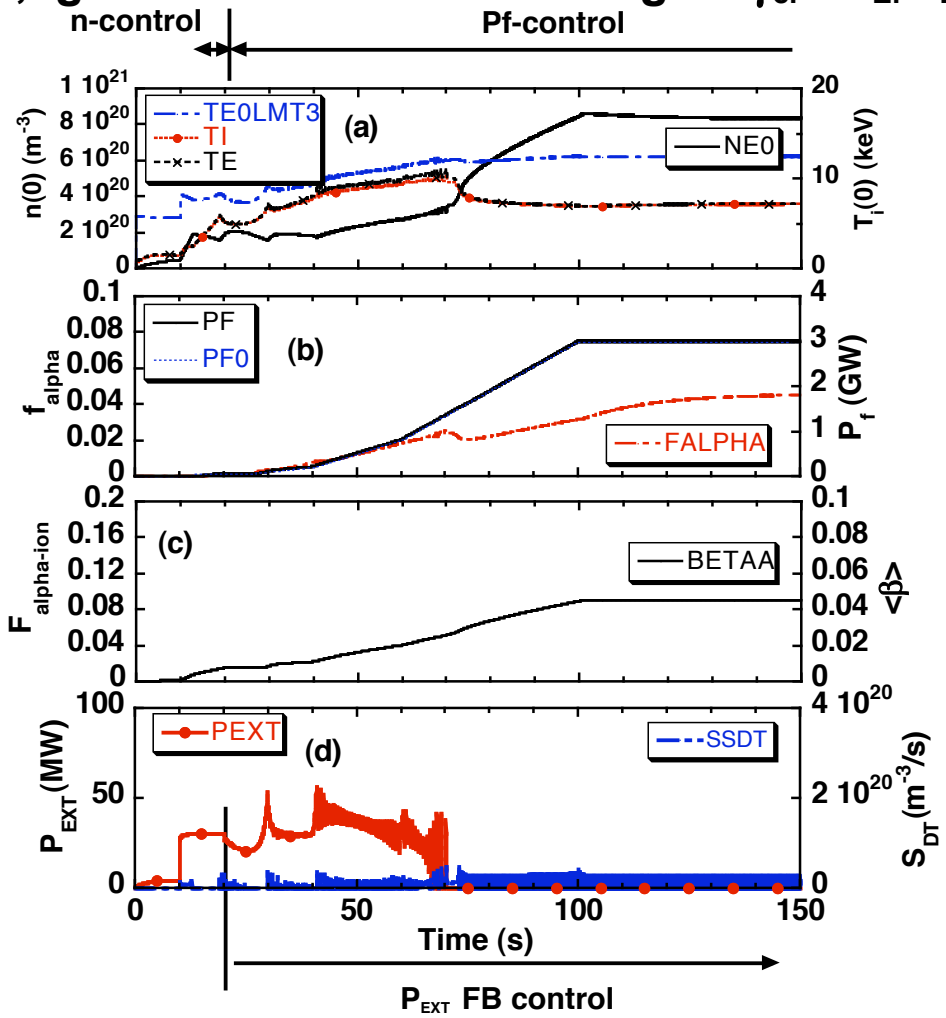


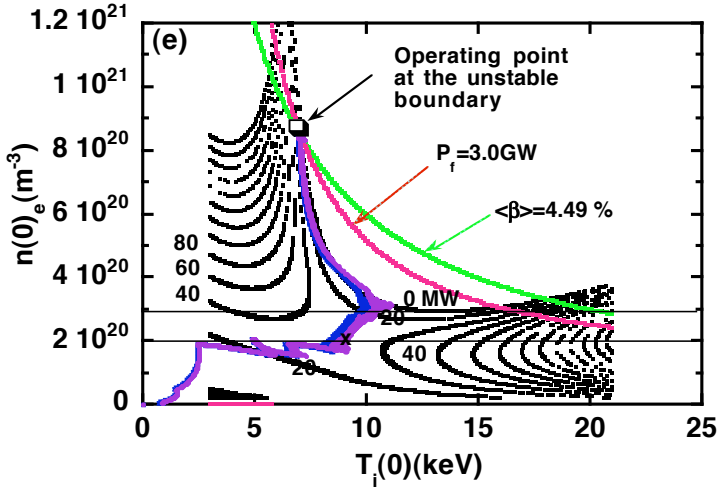


4.2 Two fluid equations: Ignition access by electron heating alone

[4.2.1] $F_{EBW\text{-}ion}$=0, $F_{alpha\text{-}ion}$ =0 (very extreme case)

Even if 100 % electron heating by EBW, 100% electron heating by alpha heating take place, ignition is achieved as long as $\gamma_{ci} = \tau_{Ei}/\tau_{Ee} = 1$ in the high density operation. T_i/T_e~0.99





Blue: Ion temperature

Purple: Electron temperature

[4.2.2] $F_{EBW\text{-}ion}=0$, $F_{alpha\text{-}ion}=0.035$

$\gamma_{ci} = \tau_{Ei}/\tau_{Ee} = 1$

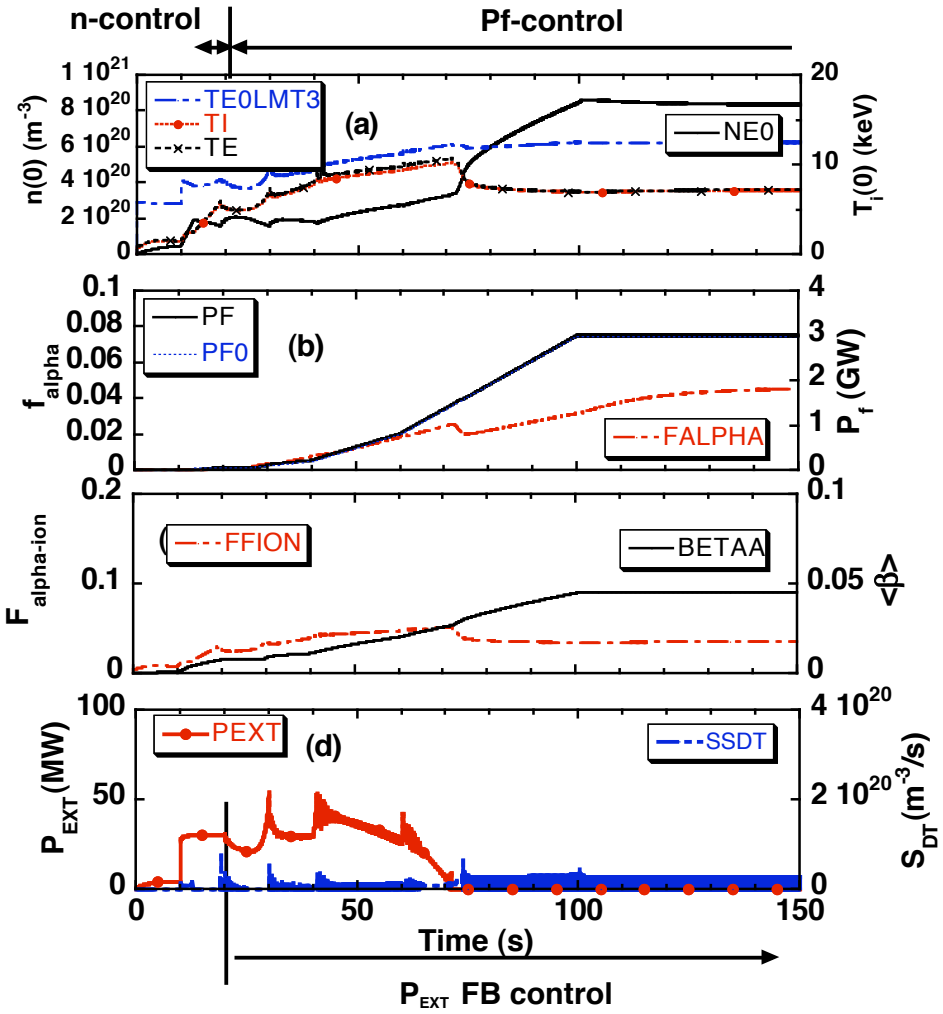
→ T_{i0}= 7.12keV, T_{e0}= 7.16 keV, T_i/T_e=0.99,

$\gamma_{ci} = \tau_{Ei}/\tau_{Ee} = 10$

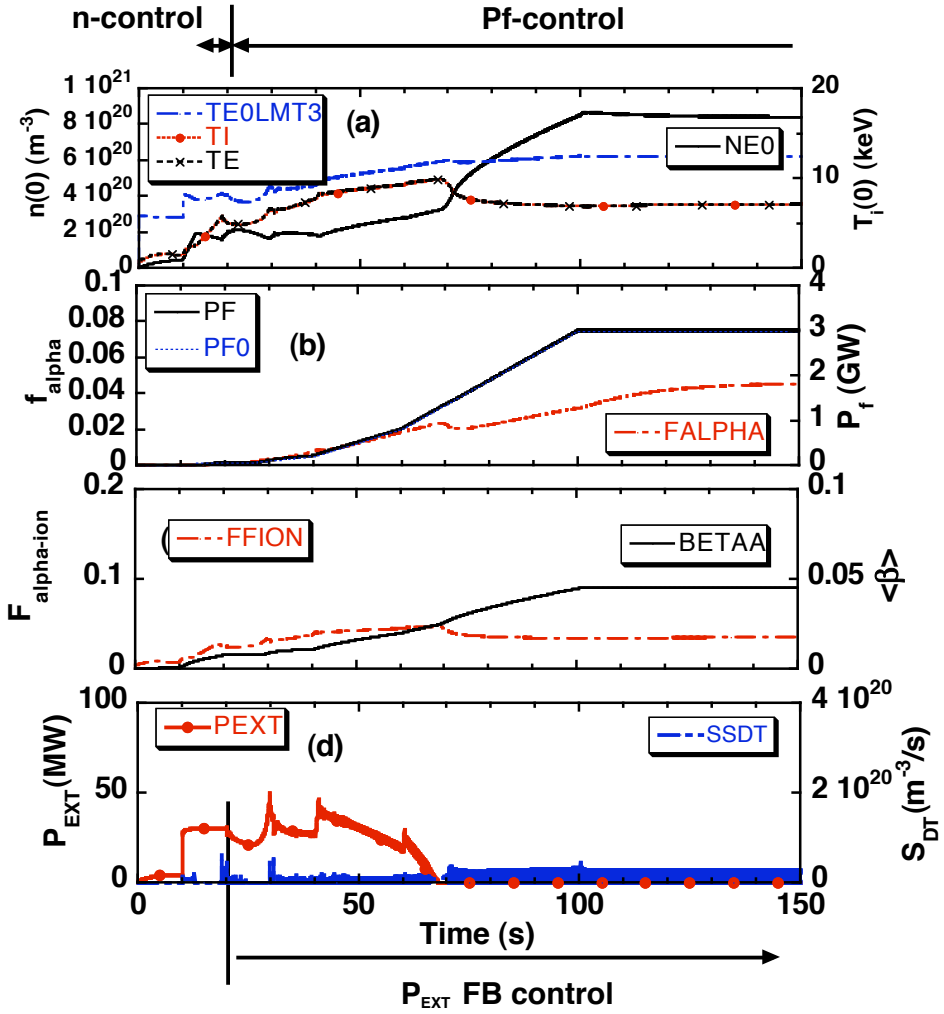
→ T_{i0}= 7.09keV, T_{e0}= 7.09 keV, T_i/T_e=1.0,

During heating phase :n(0)~2-3x10^{20} m^{-3} ≥ n_c (EBW should be used)

P_{Li}=114 MW, P_{Le}=128 MW, P_{ie}= - 90 MW



P_{Li}=19.7 MW, P_{Le}=220 MW, P_{ie}= +4.2 MW

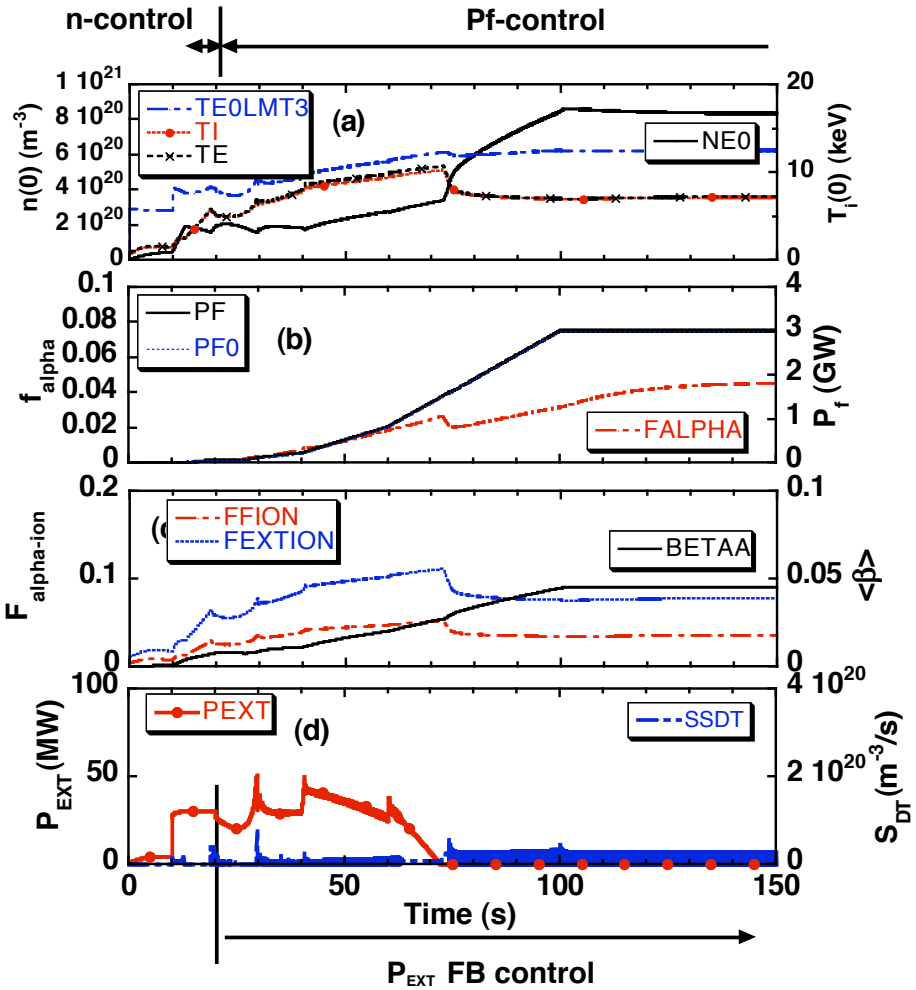


4.3. Two fluid equations: Ignition access by NBI

[4.3.1] $\gamma_{ci} = \tau_{Ei}/\tau_{Ee} = 1$

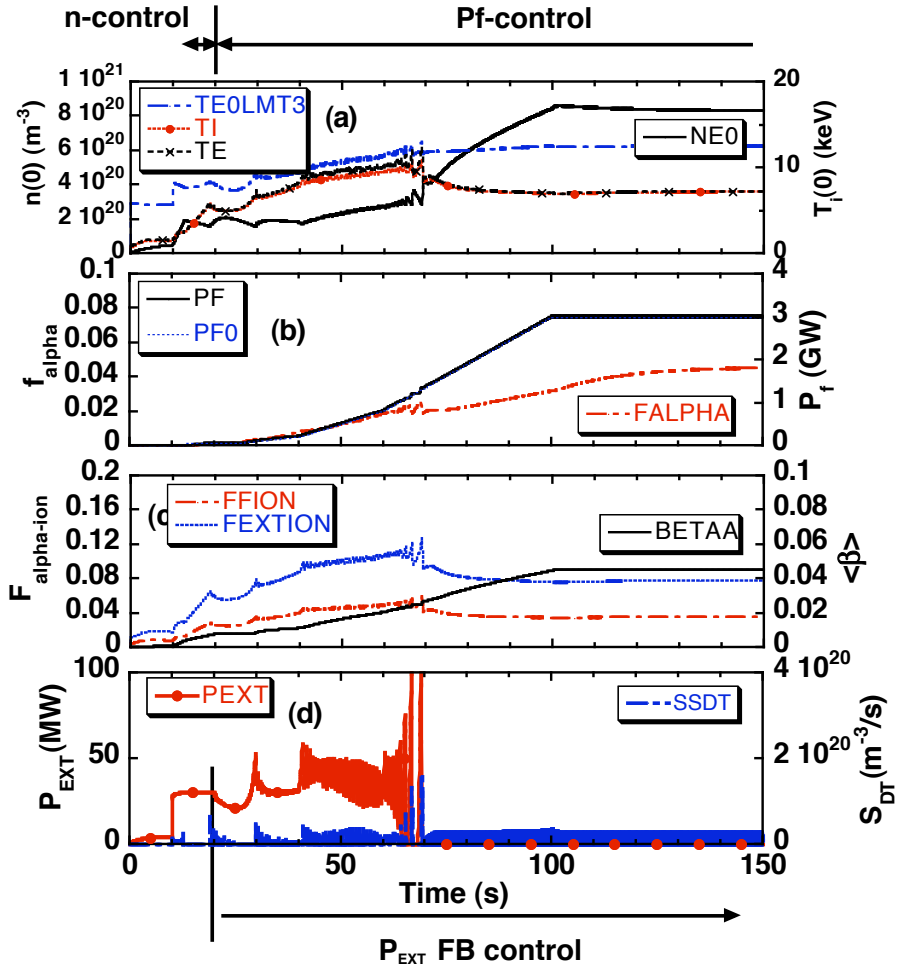
$F_{EBW\text{-}ion}$=0.1, $F_{alpha\text{-}ion}$ =0.035 → T_{i0}= 7.15 keV, T_{e0}= 7.20 keV, T_i/T_e=0.99,

During heating phase :n(0)~2-3x10^{20} m^{-3} ≥ n_c P_{Li}=114 MW, P_{Le}=129 MW, P_{ie}= - 96 MW

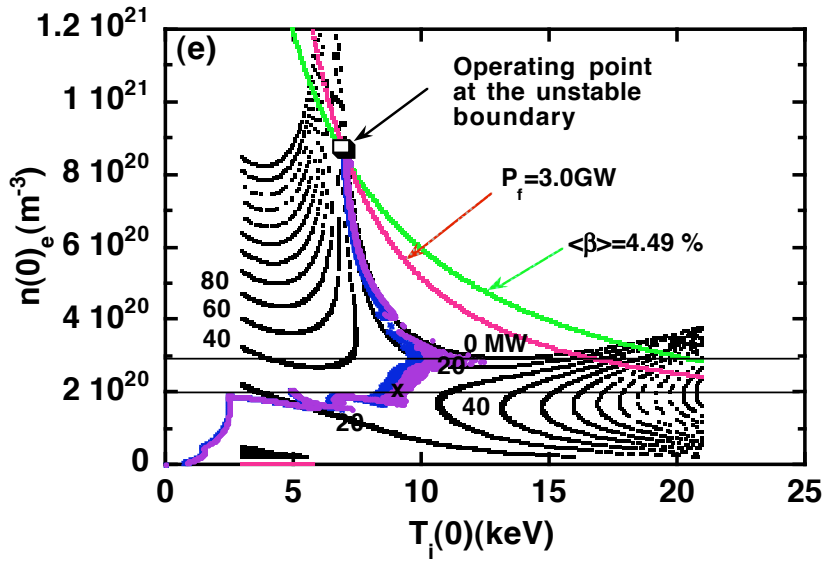


[4.3.2] If ion confinement time τ_{Ei} is shorter than the electron confinement time τ_{Ee}, ignition access becomes difficult. ($P_{EXT} \leq 100$ MW)

$\tau_{Ei}/\tau_{Ee} = 0.4$



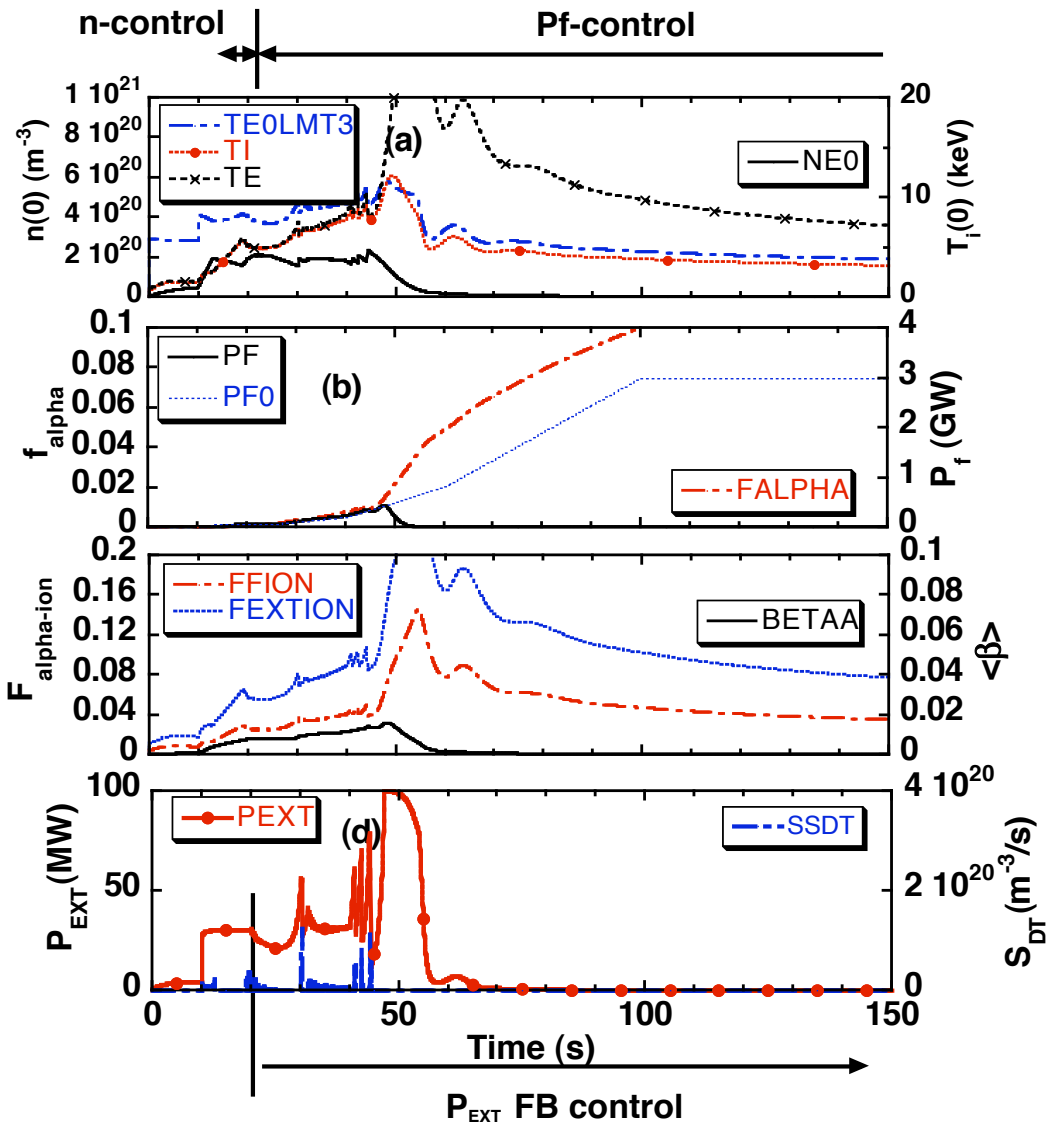
After passing the saddle point, the heating power oscillates.



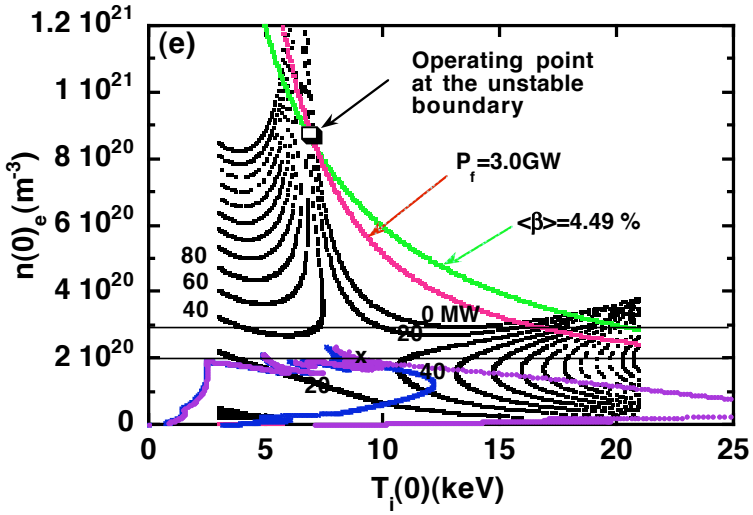
Blue: Ion temperature

Purple: Electron temperature

If electron confinement is very good as $\tau_{Ei}/\tau_{Ee} = 0.3$



Before passing the saddle point, the heating power increases the electron temperature. -> No ignition



Blue: Ion temperature

Purple: Electron temperature

5. Summary

[1] Thermally stable regime: (Higher electron temperature --> larger ion heating fraction)

ECRH (0% ion heating fraction), Alpha heating (8% ion heating fraction)

NBI (16% ion heating fraction)

- For 8 % alpha heating fraction to ions, $\tau_{Ei}/\tau_{Ee} > 2$ is necessary for ignition.
- In general, when ion confinement time is close to the electron confinement time ($\tau_{Ei}/\tau_{Ee} \sim 1$), ignition is more difficult than the case using one fluid equation.

[2] Thermally unstable regime: (Lower electron temperature-->smaller ion heating fraction)

EBW (0% ion heating fraction), Alpha heating (3% ion heating fraction)

NBI (10% ion heating fraction)

- Ignition is possible for the extreme case of 0% ion heating fraction by external heating power and 0 % ion heating by alpha heating in the case of $\tau_{Ei}/\tau_{Ee} = 1$.
- Ignition access requires the ion to electron confinement time ratio of $\tau_{Ei}/\tau_{Ee} \geq 0.4$.
- In general, ignition characteristics do not change in the high-density operation even for two fluid equations, due to strong power coupling between ion and electrons.