US-Japan Workshop on Fusion Power Plants and Related Advanced Technologies with participations of EU and Korea (22-24 Feb. 2011)

Direct Profile Extrapolation Method for the Deductive Procedure of Fusion Reactor Design

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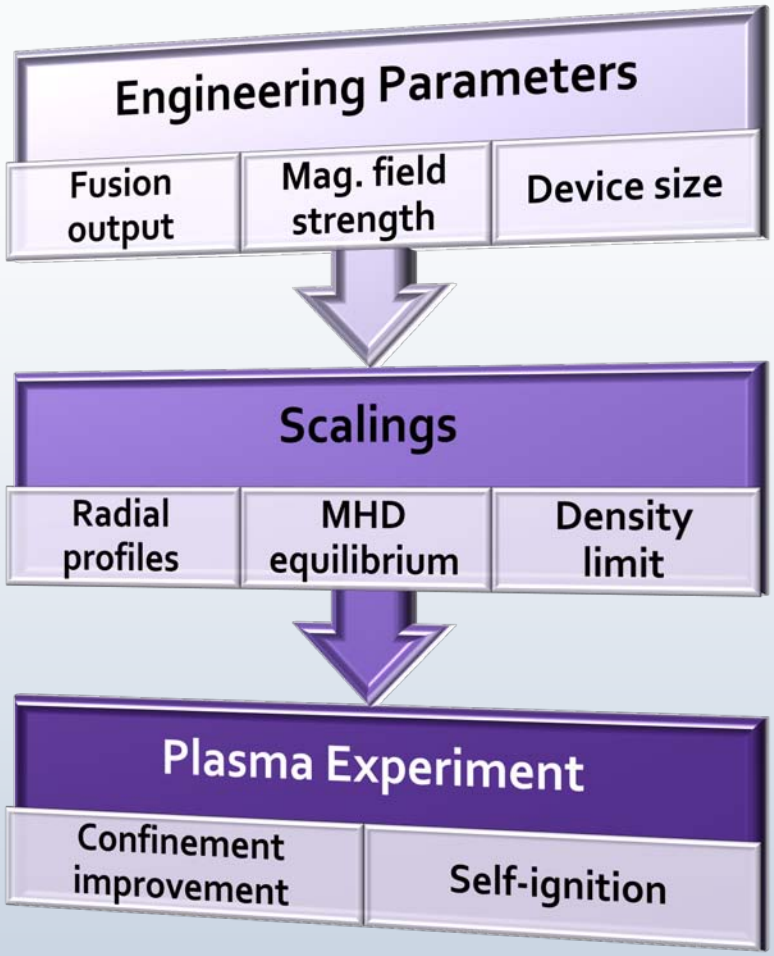
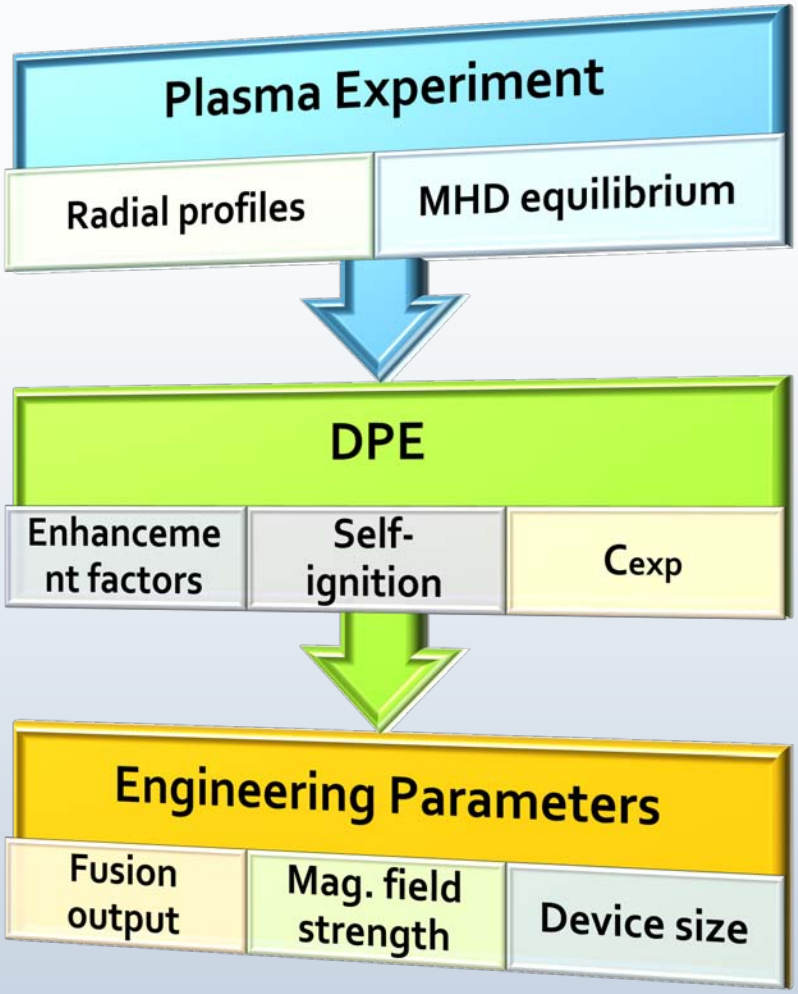
How Do You Estimate the Fusion Output?

- In general fusion reactor design activities…
  - Radial profiles: parabolic for both T and n
  - MHD equilibrium: vacuum config. is often useed in helical reactor design
  - Density: density limit scaling
  - Temperature: energy confinement scaling
  - Assumptions: $T(\rho)$, $n(\rho)$, equilibrium, n_{DL}, DL factor, $\tau_E^{scaling}$, H-factor, …
- In the DPE method proposed here…
  - Profiles and equilibrium obtained in the experiment are directly used
  - Gyro-Bohm type parameter dependence is assumed
  - Degree of freedom in determining profiles is largely reduced (= high reliability)

A New Procedure of Fusion Reactor Design



Conventional procedure (inductive approach)



New procedure using DPE (deductive approach)

DPE: Direct Profile Extrapolation

f_X : enhancement factor of X (e.g. $f_T = T_{reactor}(\rho) / T_{exp}(\rho)$, $f_n = n_{reactor}(\rho) / n_{exp}(\rho)$, $f_P = P_{reactor} / P_{exp}$)

From the definition: $f_\beta = f_T\, f_n\, f_B^{-2}$ ➔ $f_T = f_\beta\, f_n^{-1}\, f_B^{2}$ **(1)**

One can calculate the alpha heating power per unit volume by assuming f_B, f_n, and f_β

Gyro-Bohm: $\tau_E^{GB} \propto a^{2.4} R^{0.6} B^{0.8} P^{-0.6} n^{0.6}$

$n\, T\, a^2 R / P \propto a^{2.4} R^{0.6} B^{0.8} P^{-0.6} n^{0.6}$

$T \propto a^{0.4} R^{-0.4} B^{0.8} P^{0.4} n^{-0.4}$ ➔ $f_T = \gamma\, f_{a/R}^{0.4}\, f_B^{0.8}\, f_P^{0.4}\, f_n^{-0.4}$ **(2)**

γ : confinement enhancement factor

Independent of a and R

$f_{a/R} = 1$ ($f_a = f_R$)

delete f_T from Eqs. (1) and (2) ➔ $f_P = \gamma^{-2.5} f_\beta^{2.5} f_{a/R}^{-1} f_B^{3} f_n^{-1.5}$ **(3)**

Independent of a and R

Then, f_a is determined so as to satisfy

$$P_{reactor} = f_a^3\, f_{a/R}^{-1}\, f_n^2 \int_0^1 (\zeta P_\alpha' - P_B')\, (dV/d\rho)_{exp}\, d\rho = \gamma^{-2.5} f_\beta^{2.5} f_{a/R}^{-1} f_B^{3} f_n^{-1.5} P_{exp}$$

Plasma volume × f_a^3

Equilibrium in LHD

(in this study, $f_{a/R} = \zeta = Z_{eff} = 1$)

Volume integration of (alpha heating – Brems.)

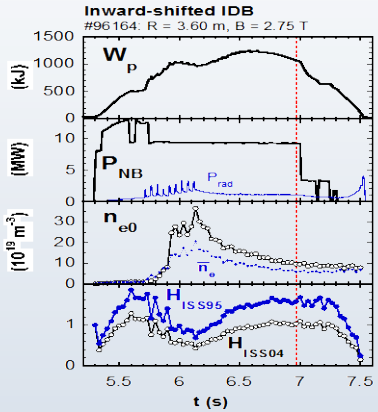
f_a (= f_R) is obtained if f_B, f_n, f_β and γ are given

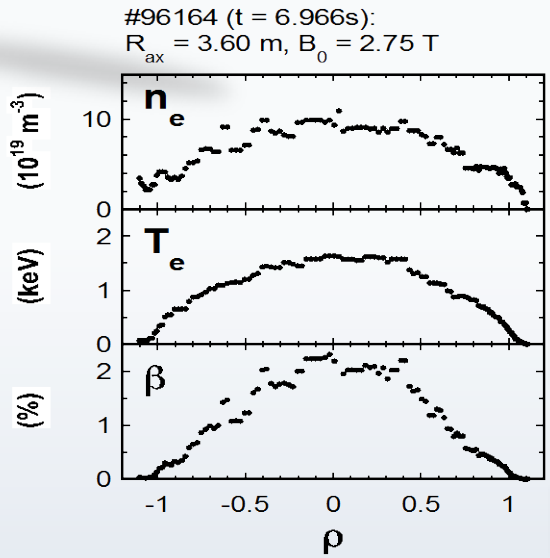
Example

Exp.

Mag. Field × f_B

Reactor

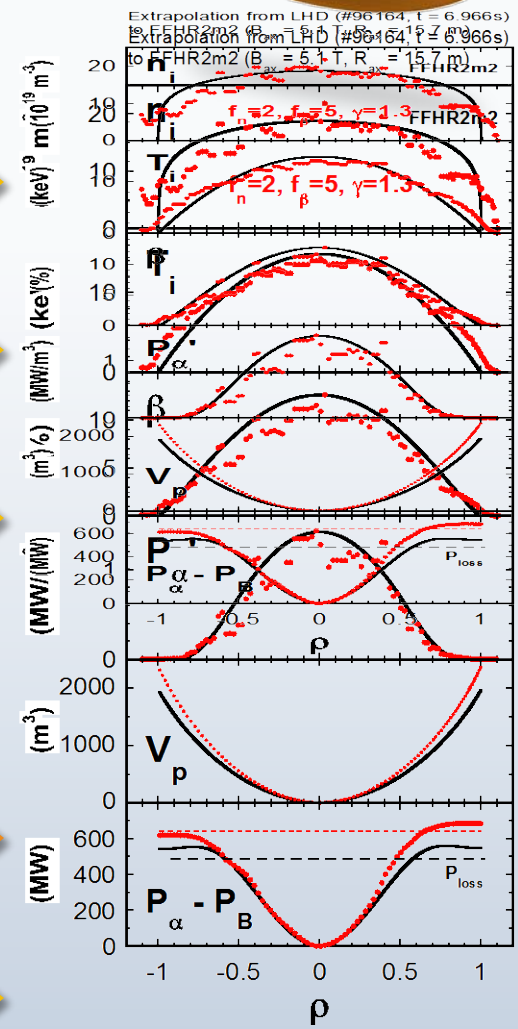




× f_n

× f_T

× f_β



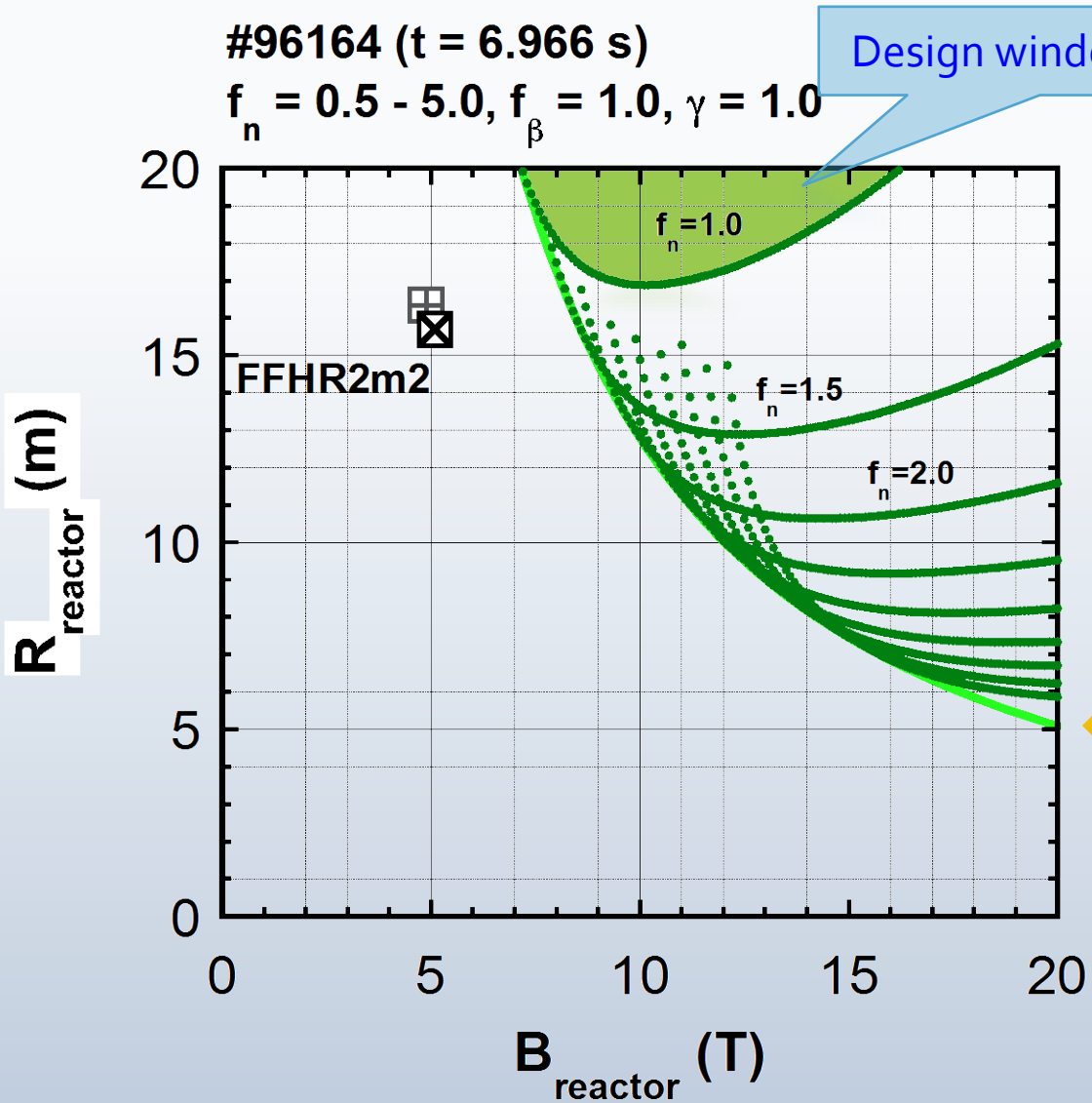
Plasma volume × f_a^3

Heating power × f_P

Confinement × γ

- f_a (= f_R) is determined so as to fulfill the power balance
- f_n=2, f_β=5 and γ=1.3 reproduce the profiles assumed in FFHR2m2

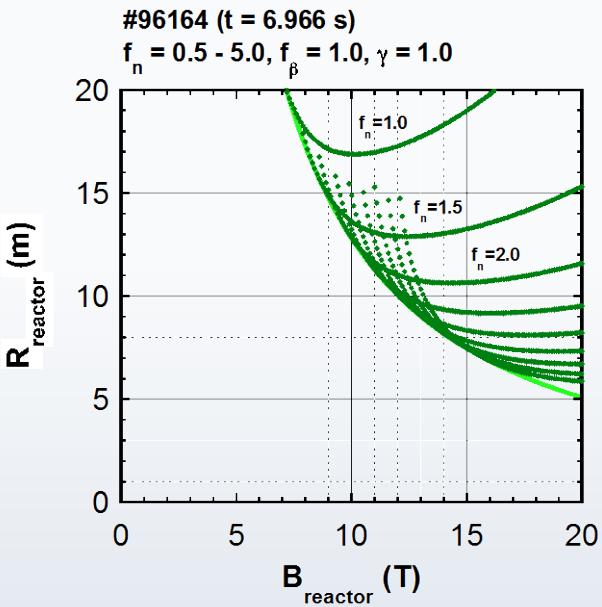
Mag. Field Strength Determines the Plasma Size

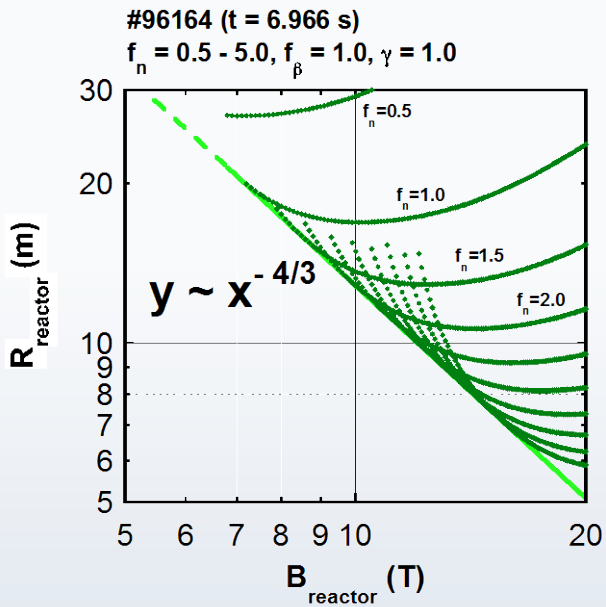


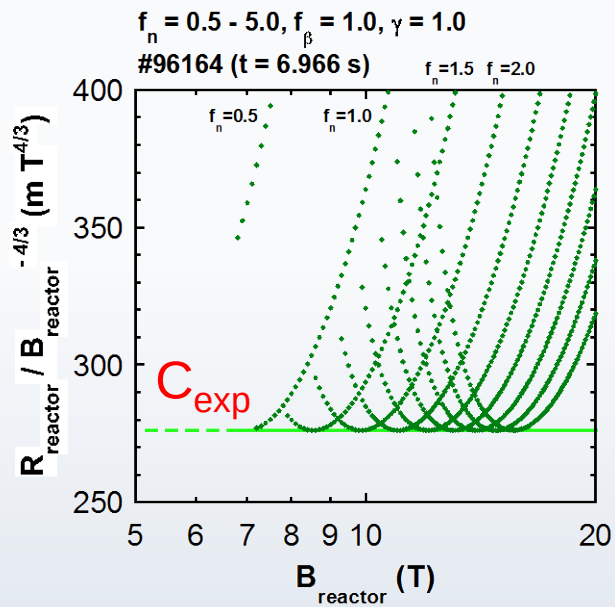
- $B_{reactor}$ is scanned for various f_n (f_β and γ are fixed to 1)
- The minimum $R_{reactor}$ is given as a function of $B_{reactor}$

What is this lower envelope?

$R_{reactor} \propto C_{exp}\, B_{reactor}^{-4/3}$







$$P_{reactor} = f_a^3\, f_{a/R}^{-1}\, f_n^2 \int_0^1 (\zeta\, P_\alpha' - P_B')\,(dV/d\rho)_{exp}\, d\rho$$

$$\propto f_a^3\, f_{a/R}^{-1}\, f_n^2\, f_T^X$$

Temp. dependence: f_T^X

$$\propto f_a^3\, f_{a/R}^{-1}\, f_n^2\, (f_\beta\, f_n^{-1}\, f_B^2)^X \propto \gamma^{-2.5}\, f_\beta^{2.5}\, f_{a/R}^{-1}\, f_B^3\, f_n^{-1.5}\, P_{exp}$$ ← Eq. (3)

$$\rightarrow f_a^3 \propto \gamma^{-2.5}\, f_\beta^{2.5-X}\, f_B^{3-2X}\, f_n^{-3.5+X}\, P_{exp}$$

f_n dependence disappears...

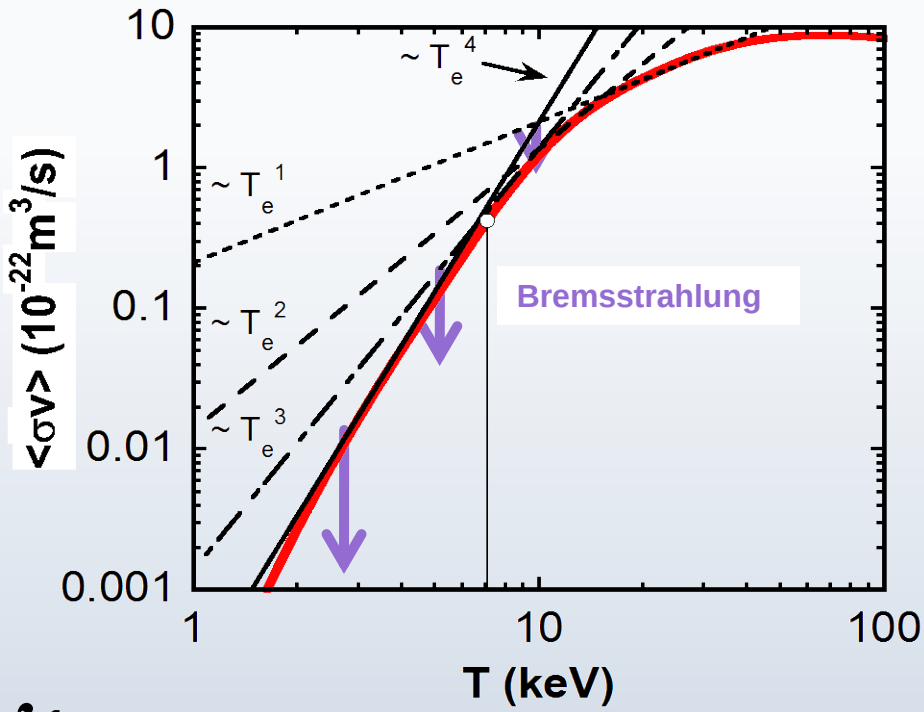
$$(X = 3.5) \rightarrow f_a \propto \gamma^{-5/6}\, f_\beta^{-1/3}\, f_B^{-4/3}\, P_{exp}^{1/3}$$

$$R_{reactor} \propto C_{exp}\, \gamma^{-5/6}\, f_\beta^{-1/3}\, B^{-4/3}$$

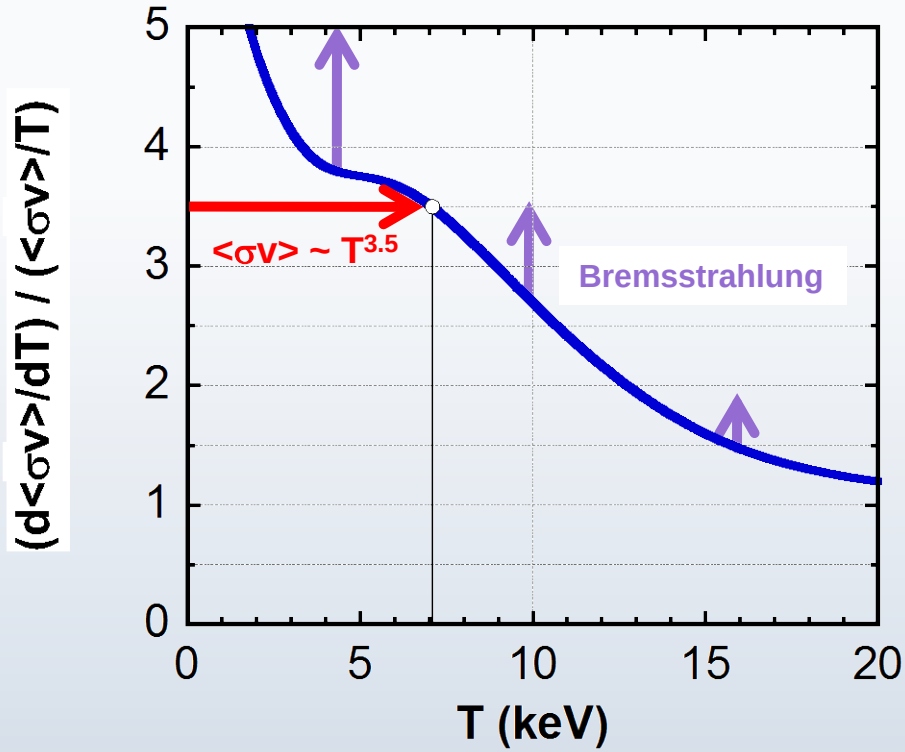
A small C_{exp} results in a compact reactor

C_{exp} is Given at a Fixed Temperature

Temp. dependence of DT fusion reaction rate
DT fusion cross-section



Temp. dependence of an index X (<σv> ~ T^X)

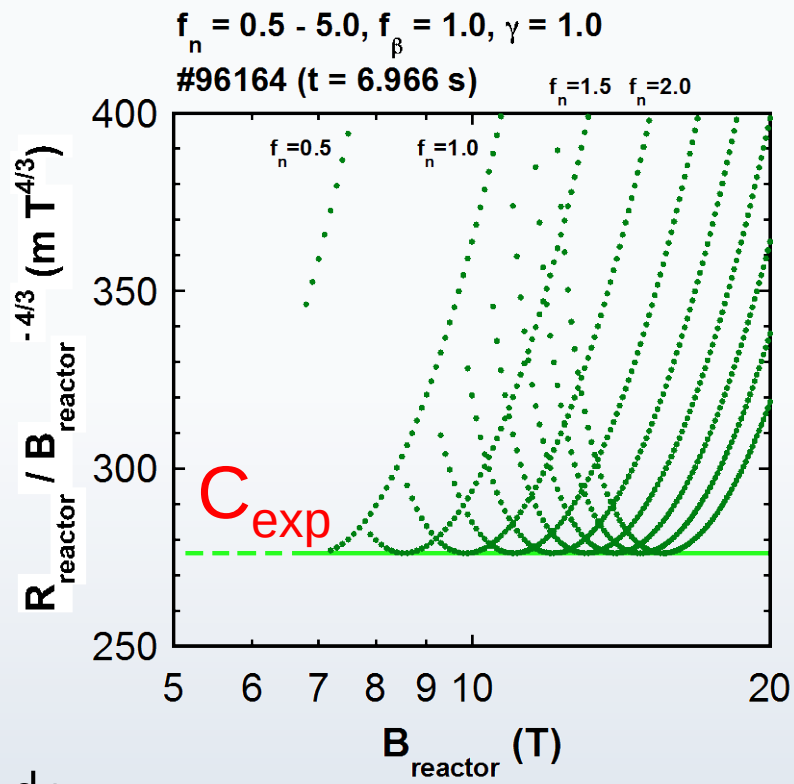


$$\int_0^1 (\zeta P_\alpha' - P_B')\,(dV/d\rho)_{exp}\, d\rho \propto f_T^X \quad | \quad \langle\sigma v\rangle \propto T^X$$

(**X =3.5** @ T ~ 7.1 keV w/o Bremsstrahlung)

Actually, the plasma size becomes minimum at **T_0 ~10 keV** (due to Brems. and volume-integration)

How to Get C_{exp}



1) Set $f_n = f_\beta = \gamma = 1$: $f_T = f_B^2$

2) Scan $B_{reactor} = f_B\, B_{exp}$:

Heating power: $f_B^3\, P_{exp}$

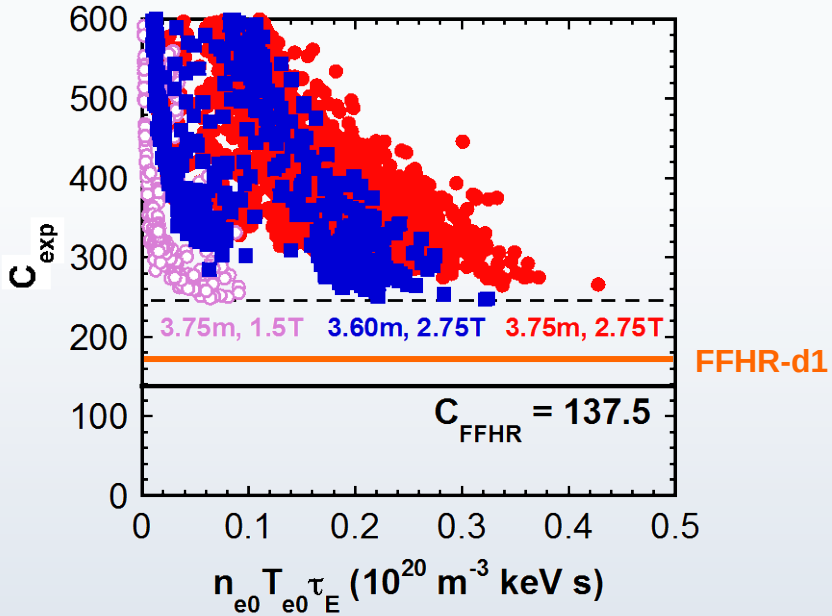
Volume-integration: $\int_0^1 (P_\alpha' - P_B')\,(dV/d\rho)_{exp}\, d\rho$

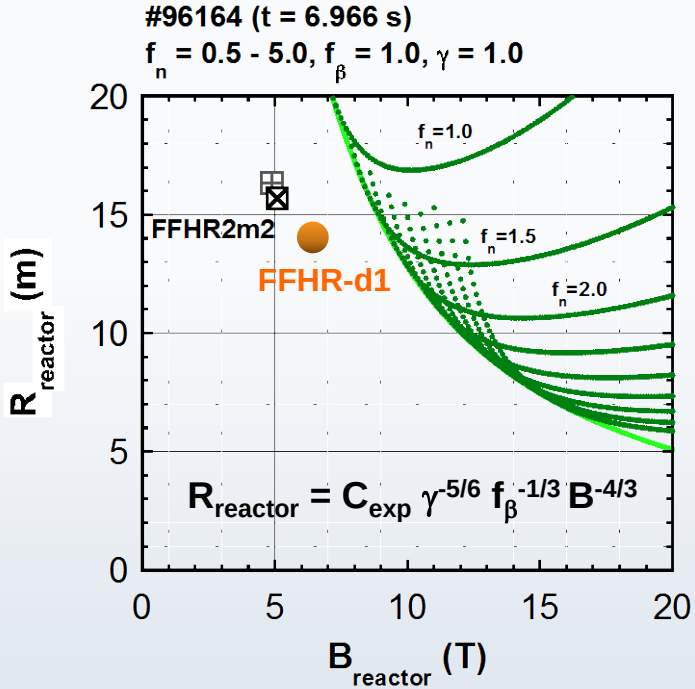
$R_{reactor}$ = [(Heating power) / (Volume-integration)]$^{1/3}$ R_{exp}

3) The minimum of $R_{reactor} / B_{reactor}^{-4/3}$ is the C_{exp}

(Note: in some cases, $f_n = 1$ might be inadequate!)

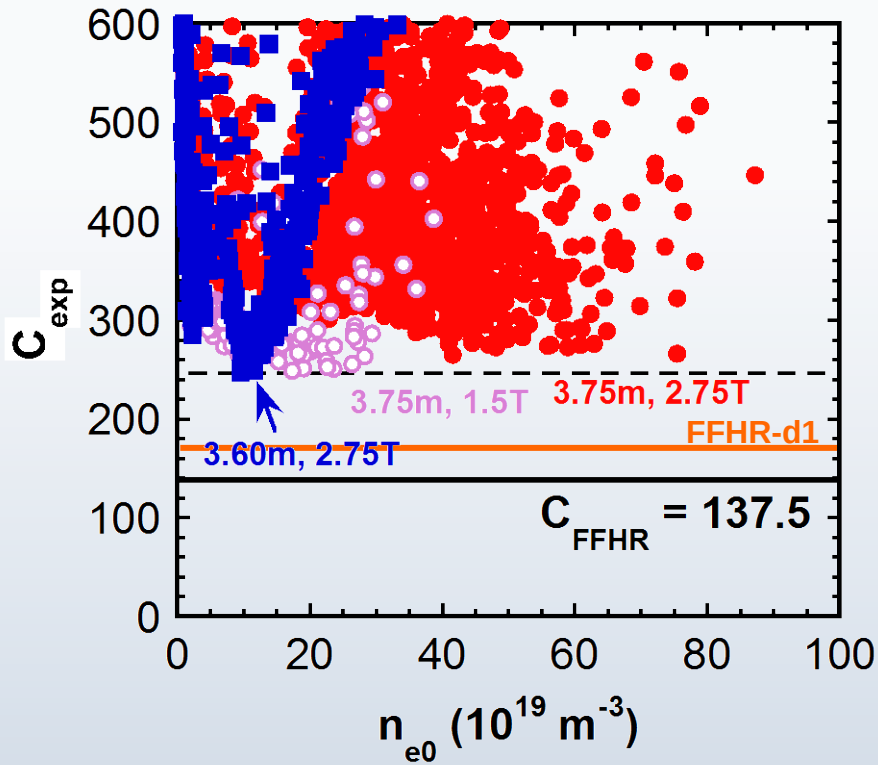
C_{exp} is a Good Measure of Plasma Performance

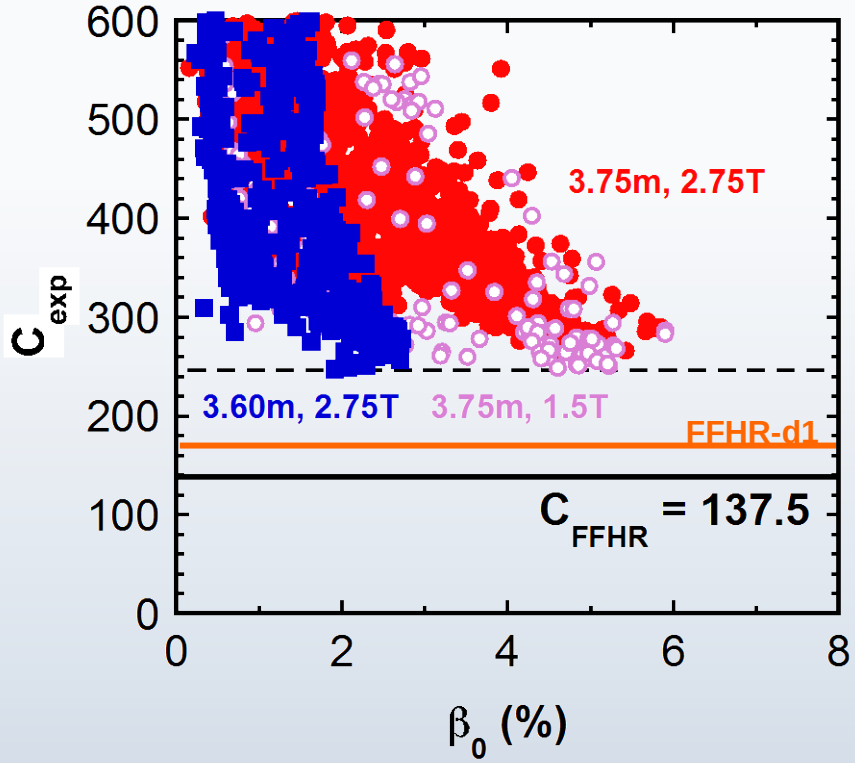




- C_{exp} can measure the plasma performance, like the fusion triple product of $nT\tau$
- Although an inverse correlation between C_{exp} and $nT\tau$ is recognized…
- The maximum of $nT\tau$ does not necessarily correspond to the minimum of C_{exp}
- $nT\tau$ is given by the averaged (or, the central) values, while the whole profiles of n and T are used to get C_{exp}
- C_{exp} that directly shows the design window is a better index than $nT\tau$!!

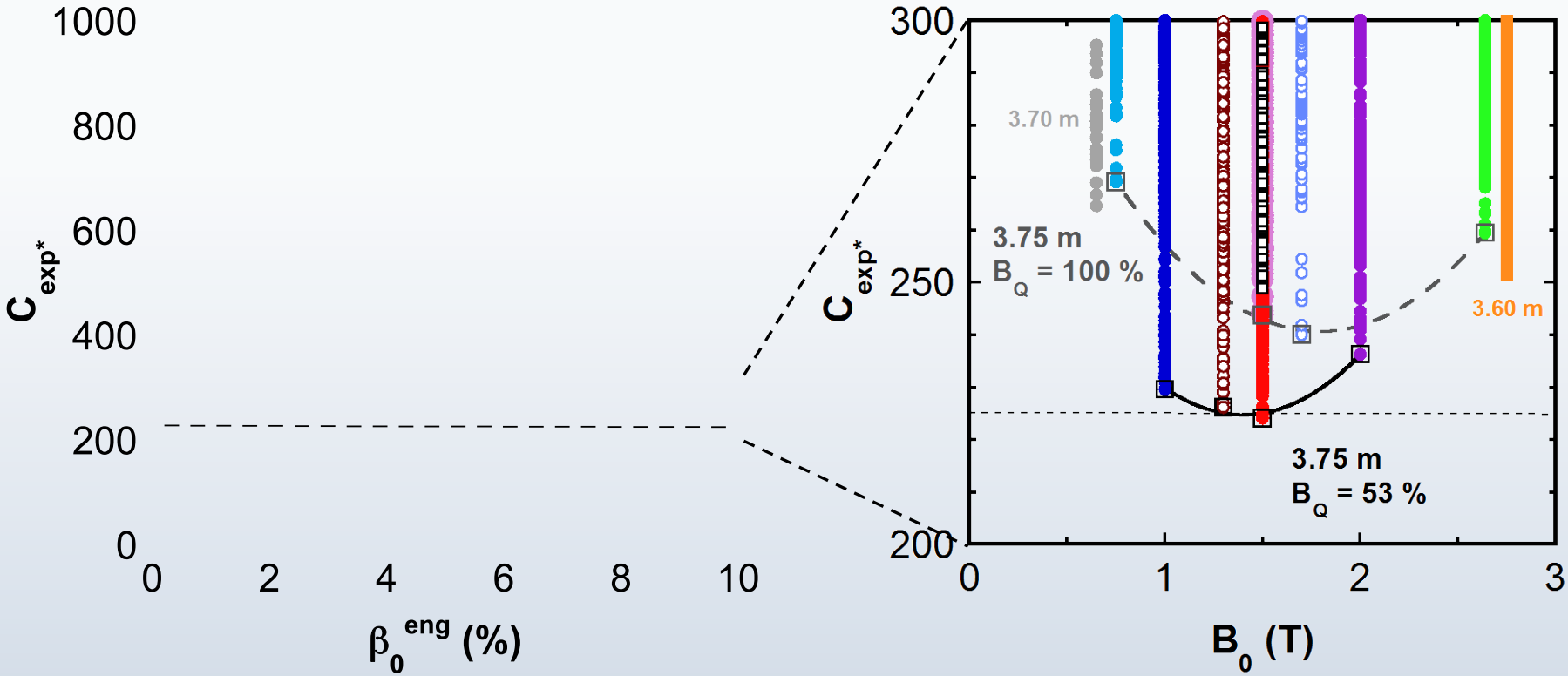
Seek the Minimum C_{exp}





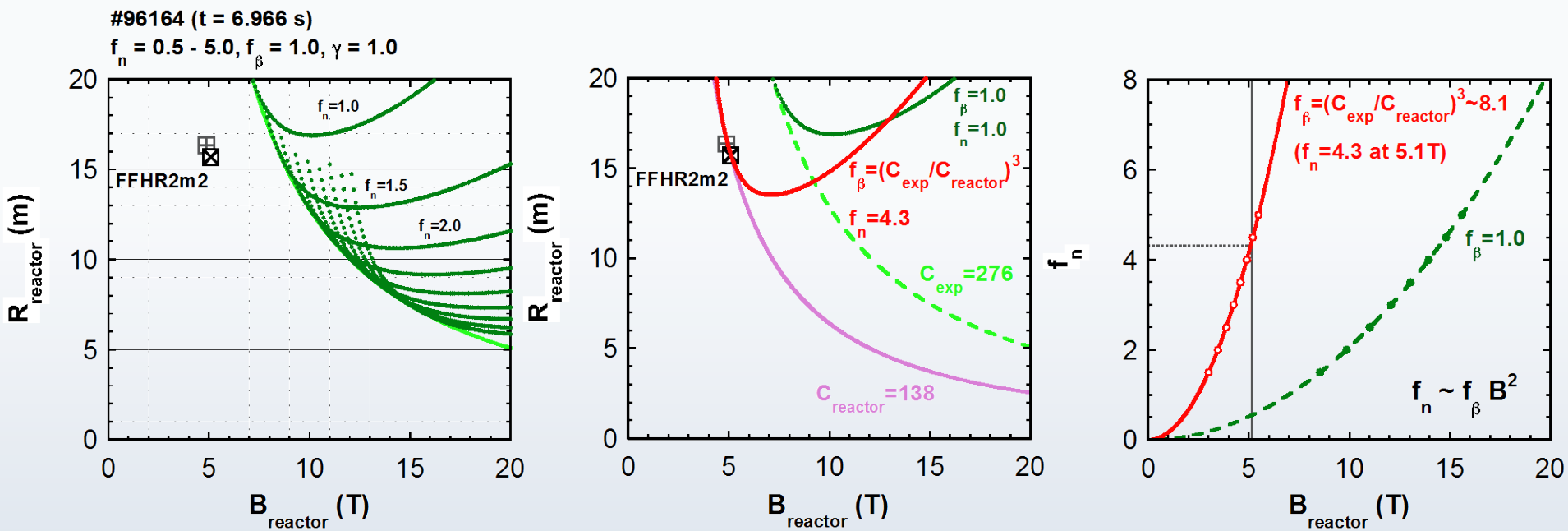
- To design a helical DEMO reactor FFHR-d1 of (R_c, B_c, $C_{reactor}$) ~ (14 m, 6.5 T, 170), we are now trying to get the minimum C_{exp} in LHD (C_{exp} ~ 225 in the 14th cycle exp.)
- How can we minimize the C_{exp}? ➔ magnetic config., high beta, high density, …

Magnetic Configuration is Important



- ✓ C_{exp*} is smaller in the vertically elongated magnetic configuration
- ✓ The optimum magnetic field strength is ~ 1.5 T

How Large Enhancement is Needed?



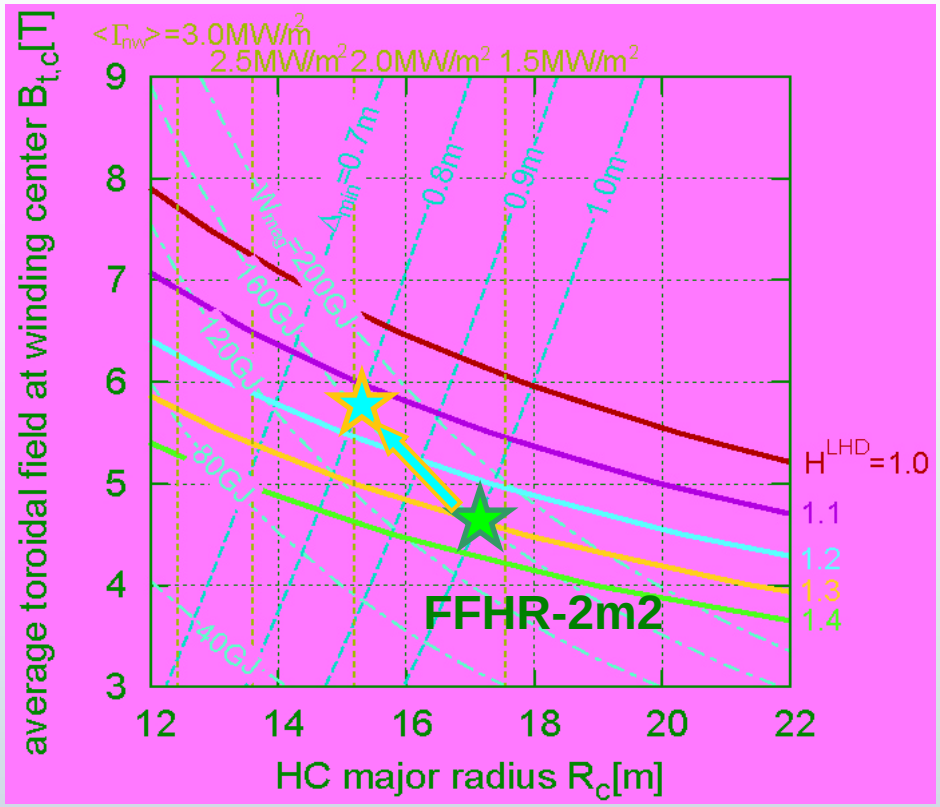
$$R_{reactor} = C_{exp}\, \gamma^{-5/6}\, f_\beta^{-1/3}\, B^{-4/3}$$

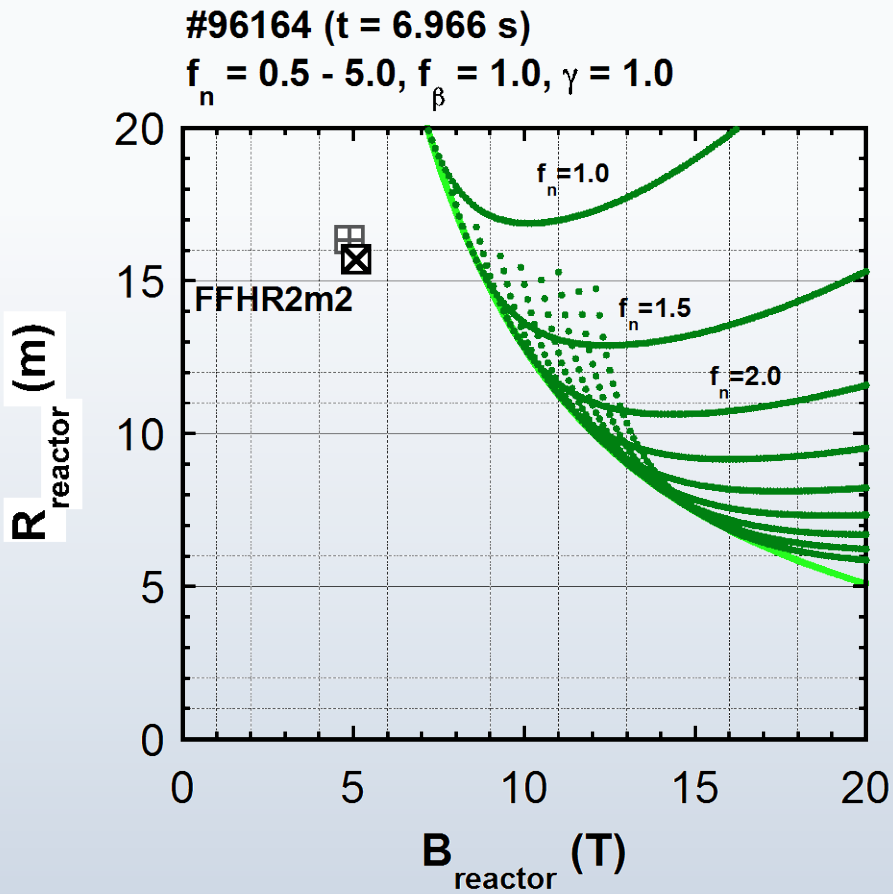
$$R_{reactor} = C_{reactor}\, B^{-4/3}$$

- If the design point is already fixed and the experimetal result is not enough, the beta should be increased with $f_\beta = \gamma^{-2.5} (C_{exp} / C_{reactor})^3$

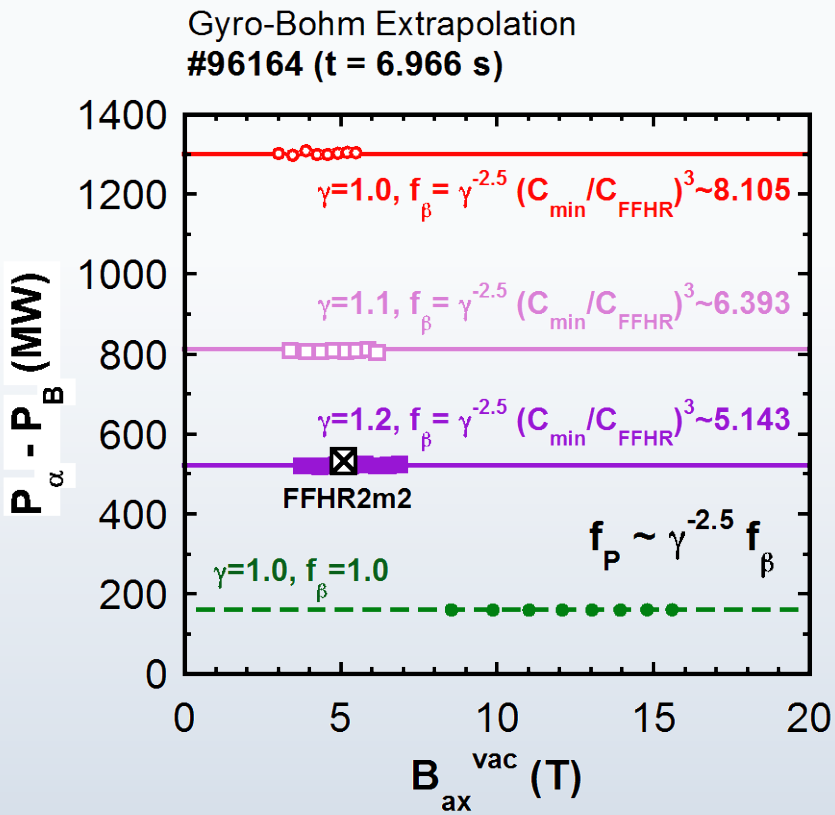
Summary

- **DPE: a new method to predict the fusion out put**
  - Using "real" profiles and equilibrium
  - Gyro-Bohm is assumed
  - f_a (= f_R) is estimated for assumed f_B, f_n, f_β, γ
- **The plasma (device) size is proportional to C_{exp}**
  - $R_{reactor} \propto C_{exp} B_{reactor}^{-4/3}$
- **Seek the minimum C_{exp}**
  - C_{exp} is a better index than $nT\tau$
- **A new procedure of fusion reactor design**
  - Deductive approach is possible with DPE


average toroidal field at winding center $B_{t,c}$[T]
HC major radius R_c[m]
$\langle\Gamma_{nw}\rangle$=3.0MW/m^2
2.5MW/m^2 2.0MW/m^2 1.5MW/m^2
Δ_{min}=0.7m
0.8m
0.9m
1.0m
W_{mag}=200GJ
160GJ
120GJ
80GJ
40GJ
H^{LHD}=1.0
1.1
1.2
1.3
1.4
FFHR-2m2


#96164 (t = 6.966 s)
f_n = 0.5 - 5.0, f_β = 1.0, γ = 1.0
$R_{reactor}$ (m)
$B_{reactor}$ (T)
f_n=1.0
f_n=1.5
f_n=2.0
FFHR2m2

ベータ値増大は出力増大を伴う



FFHRの仕様で閉じ込め改善なしならば1.3 GWの加熱パワーが必要（全核融合出力6.5 GW）

$f_\beta = 8$ で中心βは16 % (!)
ベータ限界は？

$f_P = \gamma^{-2.5} f_\beta^{2.5} f_{a/R}^{-1} f_B^{3} f_n^{-1.5}$ に $f_T = f_\beta f_B^2 f_n^{-1} = \text{const}$ を適用 ➔ $f_P \propto \gamma^{-2.5} f_{a/R} f_\beta$

◎ エンベロープでの加熱パワーは f_B と f_n には依らず、f_β に比例

◎ 閉じ込め改善度 γ によって大幅減

◎ $f_\beta \sim 5$ で磁場を低減、$\gamma \sim 1.2$ で加熱パワーを低減できればFFHR2m2は可能