Developement of System Design Code for Heliotron Reactors

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Japan-US Workshop on
Fusion Power Plants and Related Advanced Technologies
with participation of China
16-18th March, 2009 Kashiwa Campus, The University of Tokyo

Outline

1. Introduction

2. Required performance of system code

3. Consideration of simplified calculation method
   - Approximation formula
   - Inter/extrapolation

4. Summary and future work

1. Introduction

- Heliotron reactors inherently has suitable properties as a commercial plant
  - Free from any operational restriction caused by a plasma current (e.g., possibility of high density operation → low divertor heat load)
  - No need of current drive power (low recirculation power → high plant efficiency)
- LHD has achieved good plasma properties:
  - High averaged beta $<\beta>=5.1\%$
  - High density $n_e(0)=1.2\times10^{21}\mathrm{m}^{-3}$ (SDC-IDB)
- The design study of FFHR has progressed
- We have a perspective of designing a heliotron power plant

Introduction(cont'd)

- Oft-expressed disadvantages of heliotron system:
  - Narrow Space between coil and plasma
  - Low confinement due to non-axisymmetric property

    ← Can be compensated by selecting adequate design point (e.g., FFHR-2m2: enlarging plasma size compatible with a moderate construction cost)
  - Difficulties in design and construction

    ← LHD was successfully constructed and has been in operation for over 10 years
    Numerical/engineering technology has been progressed

Introduction(cont'd)

- Right now, problems lie in a design study of heliotron system are
  - Low number of the existing device and (at present) no existence of an experimental reactor prior to DEMO
  - Difficulties in an interpretation of experimental results and prediction of plasma performance (high degree of freedom in design and a requirement of complicated 3D calculation)

- These points lead to low reliability in a prediction of plant performance and difficulty in an optimization of design point.

Purpose of This Study

- For a proposal of a feasible design point, we need to find a design window with not fully optimized, but "robust" design points.
- These points should be selected with a consideration of total system design.

- Sensitivity analyses over a wide design space are needed.

→ Attempted to develop a system design code

2. Required performance of system code

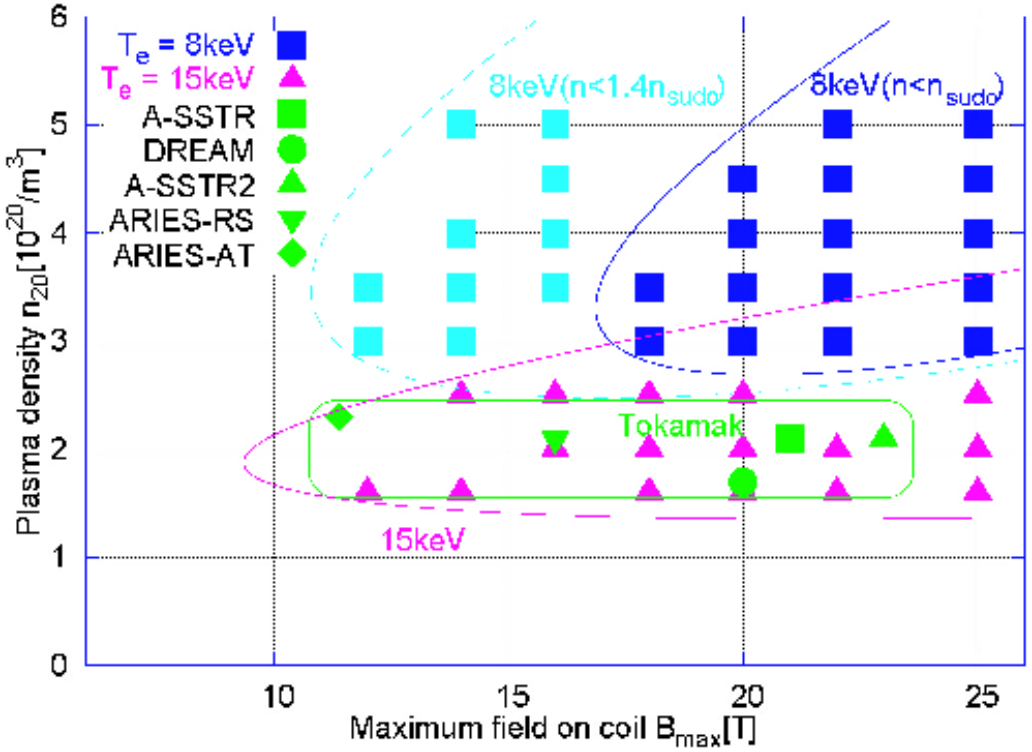
- Assume the use in design window survey through sensitivity analysis.

- Required to calculate core plasma performance, engineering design, plant performance (electric output, cost, amount of rad-waste) simultaneously.

- Whereas, computational time is desired to be as short as possible（cf. computational time of 0.864sec per one parameter set

→100,000 design points in 24 hr

6^6=46656, 3^{10}=59049）



System Code for Tokamak

- Quasi-axisymmetric properties :

  allows estimation only on a certain poloidal cross-section
- Simple toroidal magnetic field profile: $B(R)=B_{max}R_{in}/R$
- Externally controllable plasma parameters: a, κ, δ, I_p
- Empirical scaling of plasma confinement property supported by abundant experimental data:

$$\tau_E^{IPB98(y,2)}=0.0562A_i^{0.19}I_p^{0.93}R_p^{1.97}\varepsilon^{0.58}\kappa^{0.78}n_{19}^{0.41}B_t^{0.15}P_{all}^{-0.69}$$

- Fairly good prediction can be obtained by
  - 1-D design (radial build) of plasma and surrounding components
  - averaged plasma properties along a poloidal cross-section

Issues in System Code for Heliotron

- Magnetic field configuration is automatically determined by the geometry of helical and poloidal coils

➔Plasma parameters cannot be given as inputs

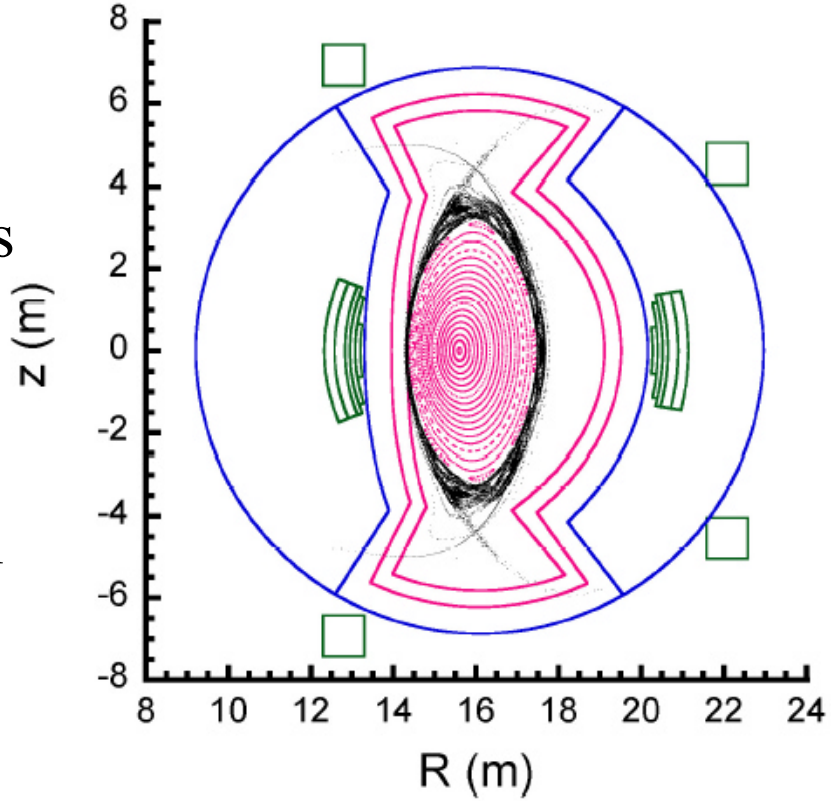
- ISS(International Stellarator Scaling) requires plasma parameters averaged over a field period:

$$\tau_E^{\mathrm{ISS95}} = 0.079\langle a_p\rangle^{2.21} R_{\mathrm{geo}}{}^{0.65} P_{\mathrm{tot}}{}^{-0.59} \bar{n}_e^{0.51} \langle B_t\rangle^{0.83} \iota_{2/3}^{0.4}$$

R_{geo} : (0,0) component of Fourier expansion of LCFS radius $R = \sum_{m,n} R_{mn}(\cos m\theta - \sin n\phi)$

$$V_p = 2\pi^2 R_{\mathrm{geo}} \langle a_p\rangle^2, \quad \langle B_t\rangle = \frac{\Phi_t}{\pi\langle a_p\rangle^2}$$

➔Need 3D equilibrium calculation



3. Consideration of Simplified Calculation

- Detailed calculation is needed for the parameters related to helical coil
  - Maximum field on coil B_{max} → Magnetic field on confinement region B_t (→Confinement performance)
  - Coil shape (R_c, a_c, m, …)→ Magnetic surface configuration (R_{geo}, $<a>$, a_{min}) (→ confinement property, blanket placement)
  - Coil shape → Stored magnetic energy W_{mag} (→ location of poloidal coil)

- They cannot be obtained by simple analytical formulae, but should be described by the geometry of helical coils

  → expected to be continuous functions of coil geometric parameters

- Can be calculated with an approximation formula or inter/extrapolation

The case of LHD optimization

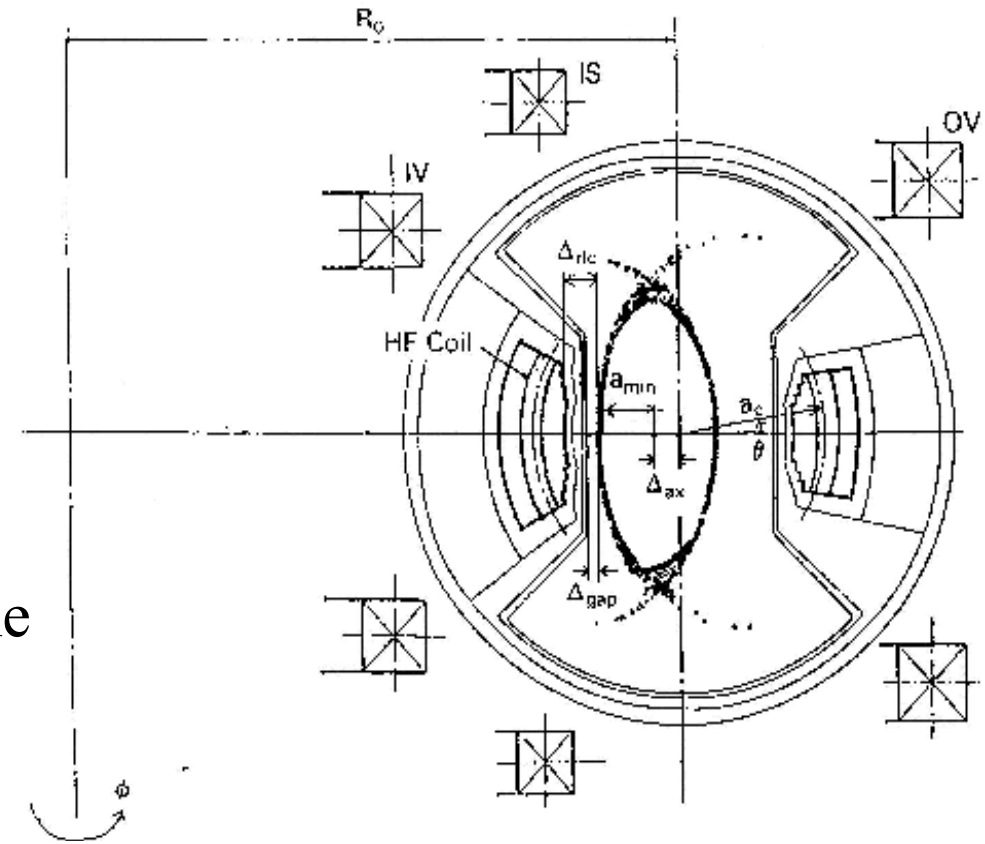
- In the design optimization of LHD, several scaling formulae proposed by Yamazaki[1] were used:

$$\frac{B_{\max}}{B_0} = 2.1\left(\frac{j_c}{40}\frac{R_c}{B_0}\right)^{0.40}\left(\frac{10}{m}\right)^{0.42}\left(\frac{\gamma_c}{1.2}\right)^{0.05},$$

$$\frac{a_p}{a_c} = 0.54\left(\frac{m}{10}\right)^{0.17}\left(\frac{\gamma_c - 0.9 - 0.44\delta}{0.3}\right)^{0.51}$$

$$\delta = \frac{\Delta_{ax}}{a_c}$$

- But these formulae don't include the effects of
  - coil cross-sectional shape
  - coil pitch modulation



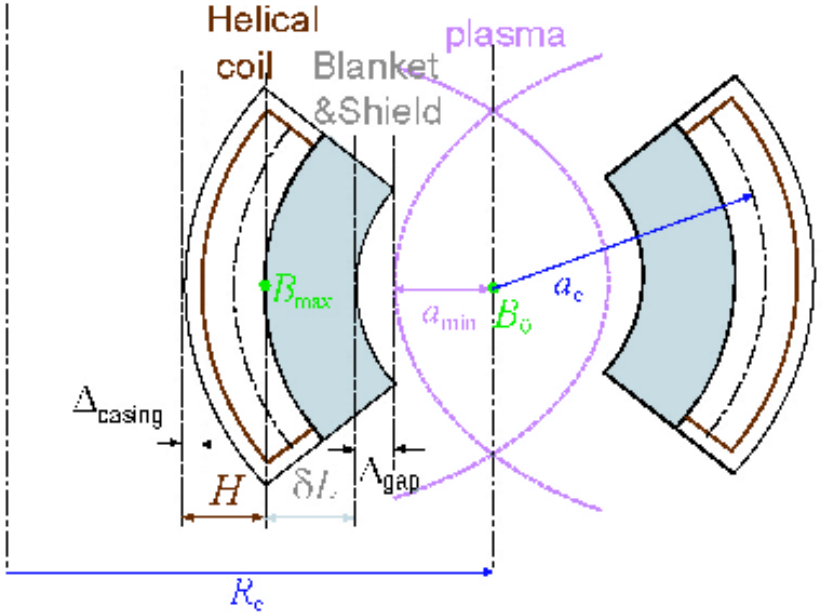
[1] Kozo Yamazaki *et al.*, Fusion Technology **21** (1992) 147.

Example of approximation formula ～magnetic field scaling～

- The maximum field on the coil B_{max} and average toroidal field $<B_0>$ are the important parameters in the reactor design.
- The ratio $B_{max}/<B_0>$ is non-dimensional and determined only by the coil geometry.

  → expected to be described with the non-dimensional parameter consisted of the parameters related to coil configuration R_c, a_c, m, α, S_c, W/H.

  (α :pitch modulation $\theta = (\ell/m)\phi + \alpha \sin[(\ell/m)\phi]$)



- Describe $B_{max}/<B_0>$ with non-dimensional parameter m, $\gamma_c \equiv m a_c/(\ell R_c)$, α, $\zeta \equiv \sqrt{S_c}/R_c$ and the parameter which gives maximum field on the infinite-length conductor with rectangular cross-section:

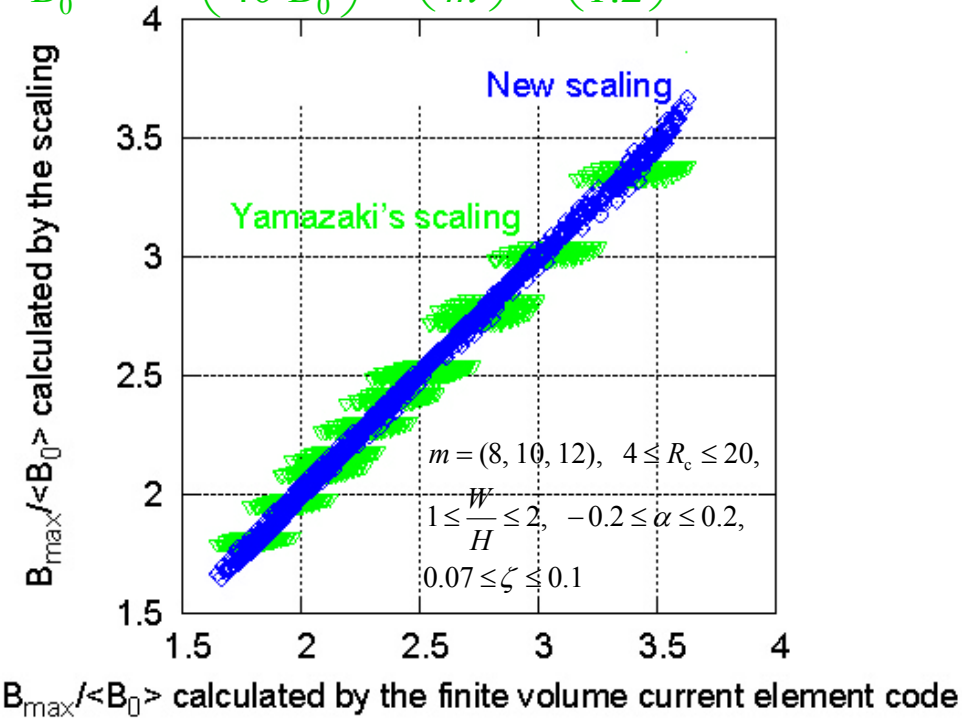
$$\xi = \sqrt{x}\left\{\ln\left(1+4/x^2\right)+\left(4/x\right)\tan^{-1}\left(x/2\right)\right\}, \quad x \equiv W/H$$

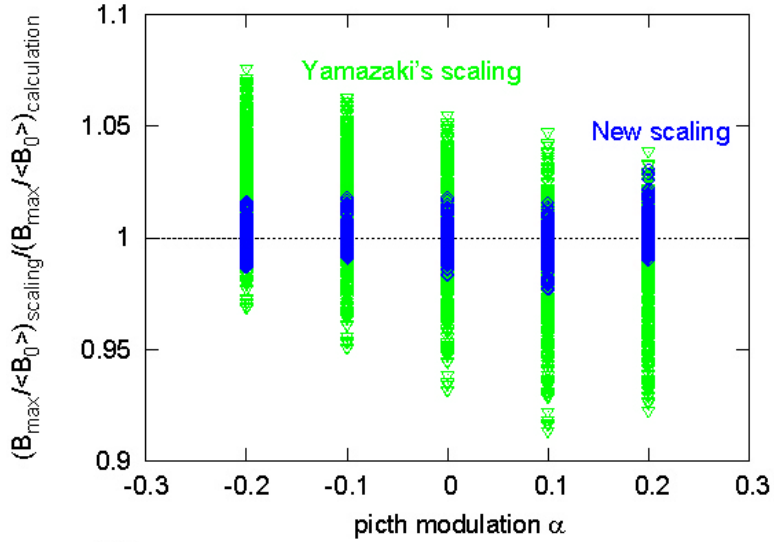
New scaling of magnetic field ratio

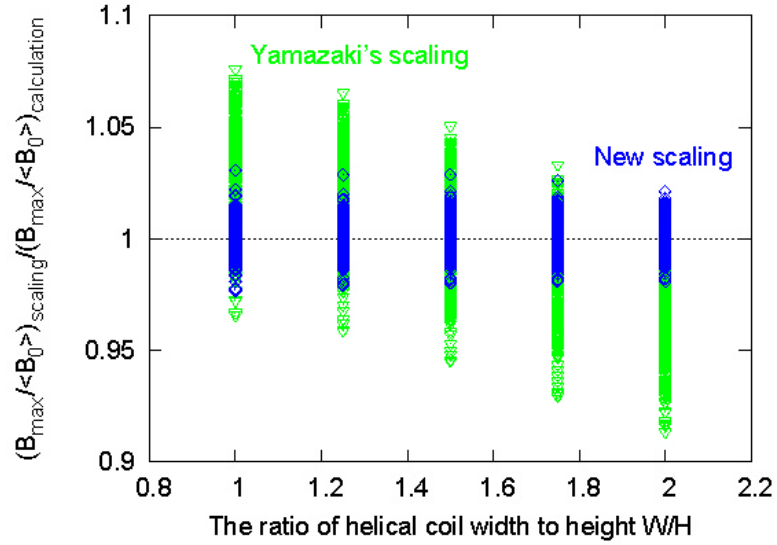
$$\frac{B_{\max}}{\langle B_0 \rangle} = 0.85(1+\alpha)^{-0.117} m^{-0.853} \gamma_c^{0.156} \xi^{0.796} \zeta^{-0.815}$$

Yamazaki's scaling law [1]:

$$\frac{B_{\max}}{B_0} = 2.1\left(\frac{j_c}{40}\frac{R_c}{B_0}\right)^{0.40}\left(\frac{10}{m}\right)^{0.42}\left(\frac{\gamma_c}{1.2}\right)^{0.05} \propto m^{-0.82}\gamma_c^{0.05}\zeta^{-0.8}$$







*Presented at 18th TOFE in San Francisco

Calculation of vacuum magnetic field

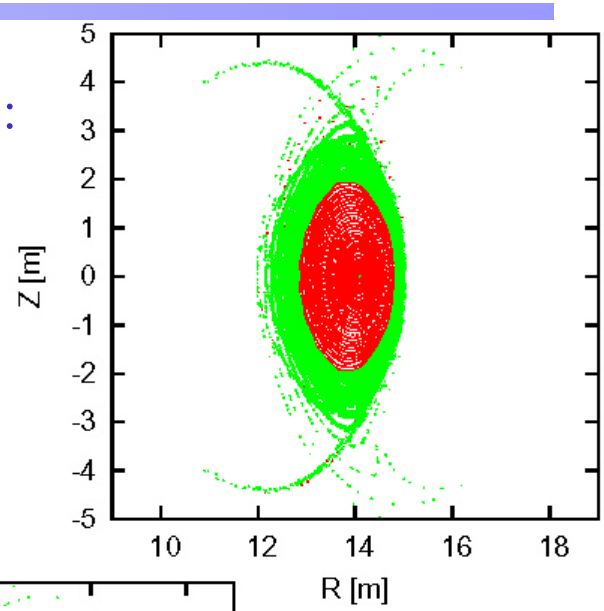
- We need several parameters of equilibrium magnetic field;

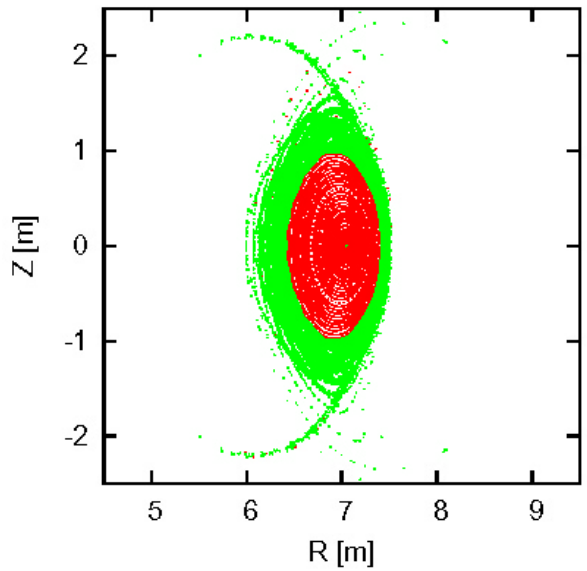
  $<a_p>$, R_{geo}, a_{min}, … etc.

- If two coils are similar to each other, generated magnetic surface configurations including ergodized layer are also similar to each other.

- Database generated by the calculation with several points of γ_c, α, ζ, W/H, R_{ax} for baseline design (e.g., R_c=10m, I_c=1MA) can yield values ($<a_p>$, a_{min}, V_p, R_{geo}, Φ_t) by inter/extrapolation.

FFHR-2m1:
R_c=14m

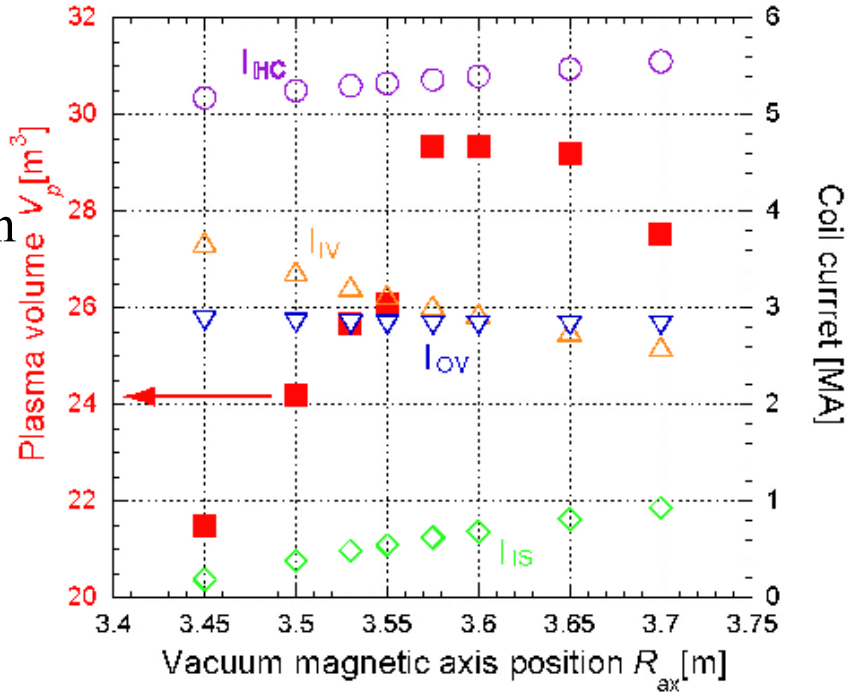




1/2 scale:
R_c=7m,
dimension:1/2
current:1/4

Effects of poloidal coils

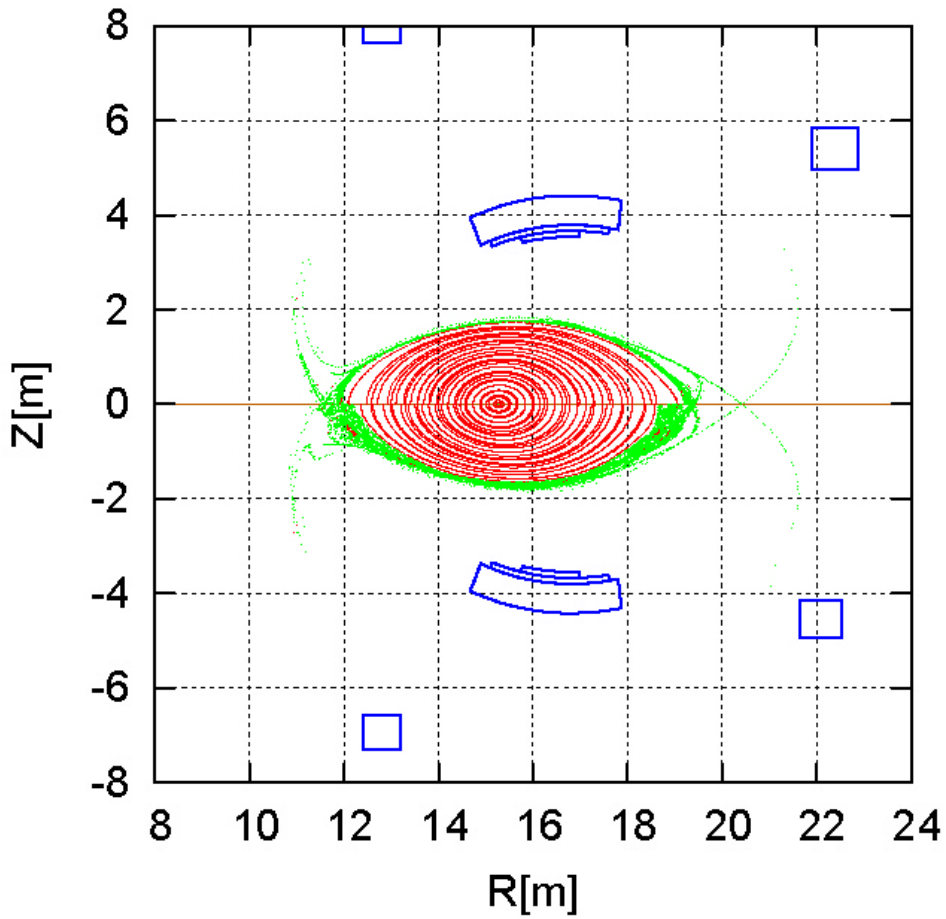
- In the case of LHD, the following 3 conditions were considered to determine poloidal coil currents:
  - Adjustment of the position of vacuum magnetic axis R_{ax}
  - Adjustment of BQ value (the ratio of cancellation of the quadrapole field generated by helical coils)
  - Minimization of a leakage field at R=2.5R_c (location of NBI system)
- LHD also has discontinuous point in plasma parameters



- For a DEMO/commercial reactor, the use of 2 pairs of poloidal coils has been considered but these location has not yet been fixed.

Placement of poloidal coils

- The structure of equilibrium magnetic field is *strongly* affected by location and current of poloidal coils.



After modification:

R_{IV} / R_{OV}= 12.8 / 22.4 m
Z_{IV} / Z_{OV}= 8.0 / 5.4 m
I_{IV} / I_{OV}= 11.8 / 18.9 MA
$\langle a \rangle$=2.465m, V_p=2050m^3

Initially proposed for FFHR-2m2:

R_{IV} / Z_{OV}= 12.8 / 22.1 m
Z_{IV} / Z_{OV}= 6.8 / 4.5 m
I_{IV} / I_{OV}= 14.7 / 17.9 MA
$\langle a \rangle$=2.225m, V_p=1570m^3

Statistical analysis (Wendelstein 7-X)

- Wendelstein team has achieved to recover equilibrium magnetic configurations of W7-X (*including finite-beta effect*) by function parameterization with a quadractic/cubic polynomial [2, 3]:

$$y_\ell = \sum_{i=0}^{N_m}\sum_{j=0}^{i} p_{ij} x_i x_j, \quad y_\ell = \sum_{i=0}^{N_m}\sum_{j=0}^{i}\sum_{k=0}^{j} p_{ijk} x_i x_j x_k$$

- However, even in this case, the degree of freedom is 6 (current ratios).
- In our case, the degree of freedom is over 10!

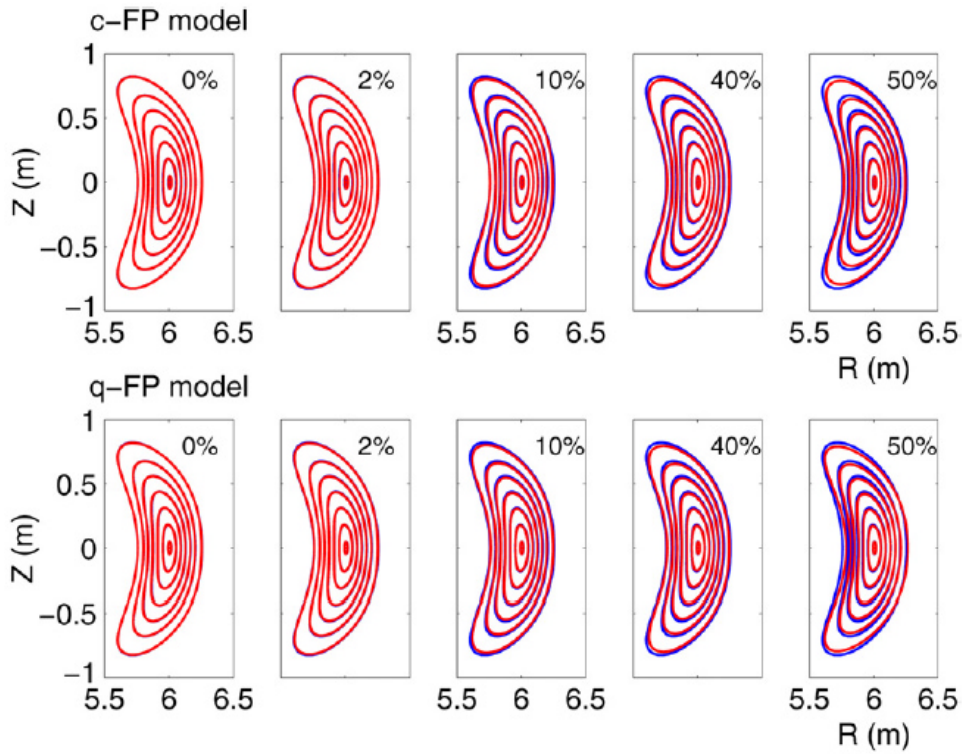


Figure 9. Flux surface recovery in the bean-shaped plane for equilibrium case 6385 in the database. Quoted are percentage noise on $J(s)$. Blue: VMEC surfaces; red: FP-recovered surfaces.

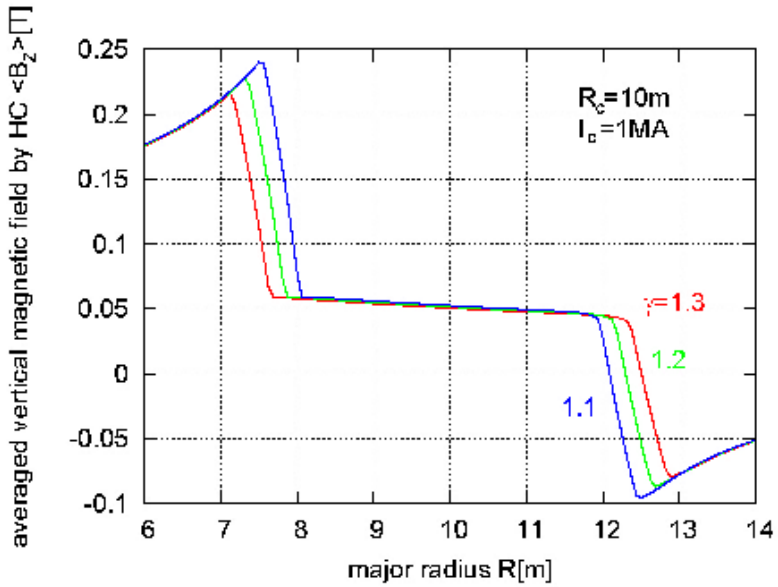
[2] A. Sengupta *et al.*, Nucl. Fusion **44** (2004) 1176.
[3] A. Sengupta *et al.*, Plasma. Phys. Control. Fusion **49** (2007) 649.

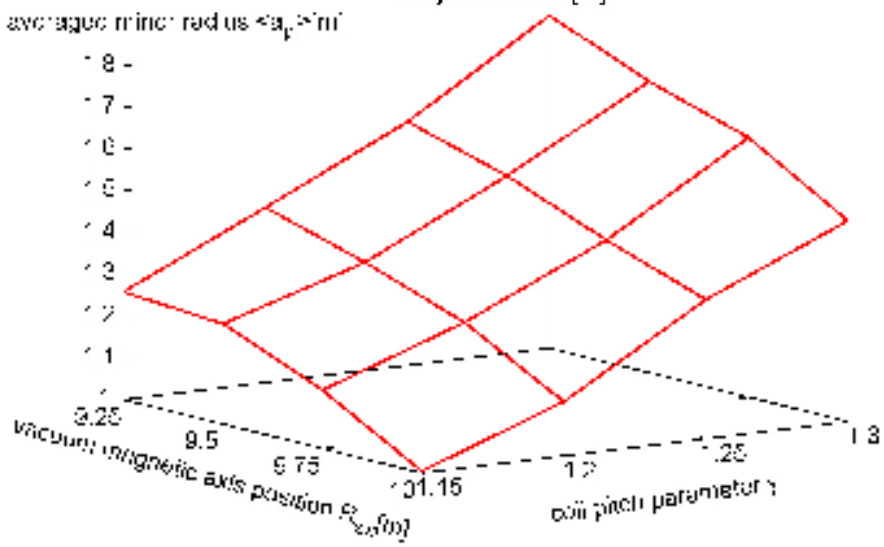
Optimization is really needed?

- Ideally, parameterization of equilibrium field properties with “optimized” property of poloidal coils for each configuration of helical coils is desired.

  …but, what is the judgment condition of optimization?
- In case of LHD and FFHR, vertical field profile without high-order components tends to generate “good” equilibrium field structure.

- There is a possibility to establish method that can recover equilibrium magnetic field structure with a reasonable accuracy for the use in parameter scanning.



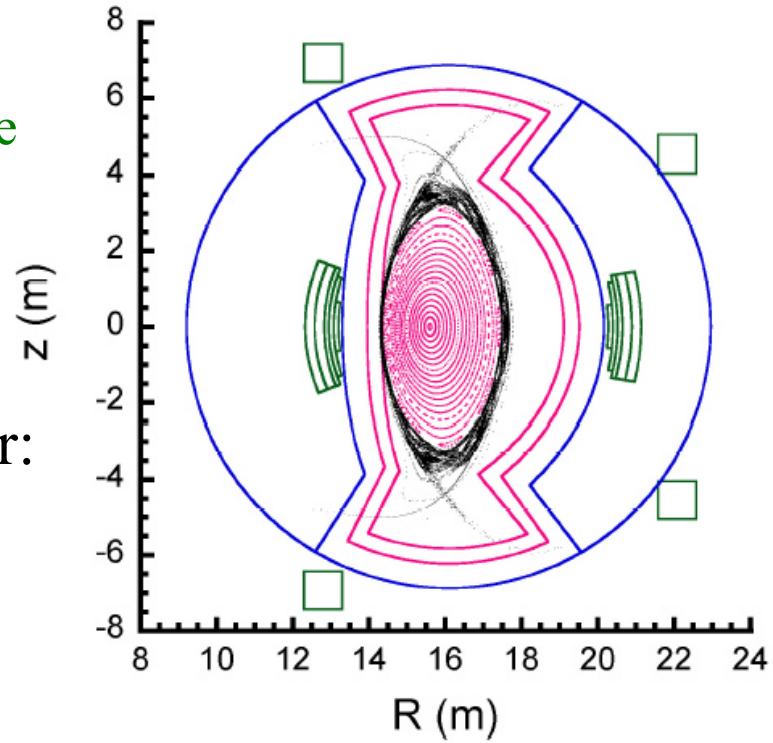


4. Summary and future work

- System design code for heliotron reactors is being developed. Simplified calculation methods expected to enable sensitivity analyses over a wide design space.
- Immediate issue is to establish a method that can recover equilibrium magnetic field structure with a reasonable accuracy.
- To achieve high reliability, we will need to
  - improve liability in estimation of plasma performance by
    - refining ISS scaling law (considering of dependence on magnetic axis shift and density / temperature profile)
    - considering finite β effect
  - develop a simple evaluation method for TBR, maintenance, and operation scenario.
- Operation regime with high robustness (that isn't so much affected by a model ambiguity) is important for assured progress towards a commercial reactor, instead of the (locally) optimized design point.

Other Items to be considered

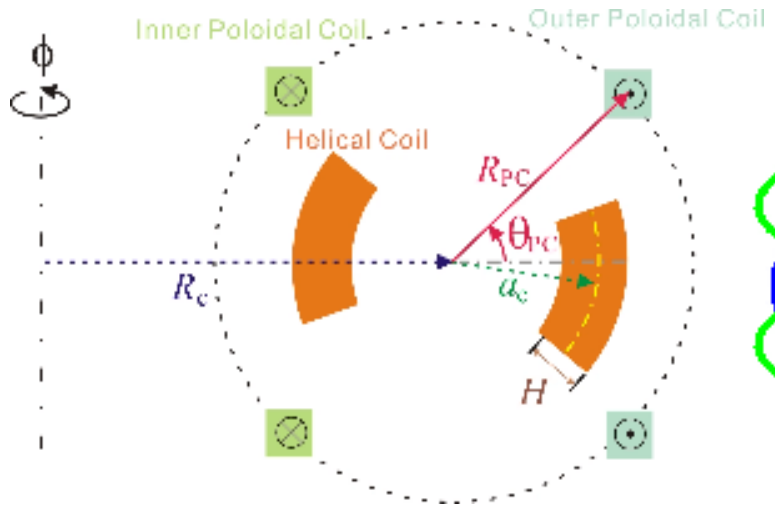
- If equilibrium field structure is successfully recovered by a fitting formula or database, we can evaluate
  - Space for blanket, shield and coil supporting structure
  - The shape of VV (balance with divertor strike point)
  - Stored magnetic energy and stress on each coil



- To design power plant, we also need to consider:
  - Sufficient TBR achievement
  - The effect of frequency and required time of maintenance on plant availability
  - Plant power balance in transient phase (e.g., plasma lump-up)

Placement of poloidal coils

- The location of poloidal coil (PC) needs to be fixed to carry out equilibrium magnetic surface.
- PC current is determined by dipole component (BD) and quadrupole component (BQ).
- Magnetic field generated by helical coil can be calculated by inter/extrapolation of tabulated data with sufficient accuracy.



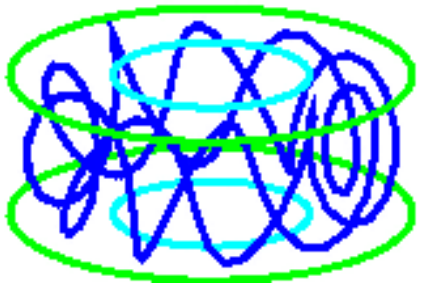


Table: Magnetic field generated by helical coil with R_c=10m. I_c=1MA

	BD [T]	BQ [T/m]	$B_z(R=0)$ [T]	$B_z(@2.5R_c)$ [Gauss]	$B_z(@5R_c)$ [Gauss]
γ_c =1.25, ζ=0.15 W/H=0.095	0.58939	-0.3277	1.2357	-508.12	-54.428
linear interpolation	0.58954	-0.3280	1.2356	-508.15	-54.431
γ_c=1.5, ζ=0.06, W/H=2.2	0.55277	-0.2998	1.2273	-515.62	-55.140
linear extrapolation	0.55116	-0.2996	1.2272	-515.73	-55.150

Placement of poloidal coils

- Parameters related to the magnetic field configuration can also be estimated by inter/extrapolation of tabulated database.
- The volume enclosed by LCFS strongly depends on the PC current and location. →need further optimization

	LHD R_{ax}=3.9m		LHD R_{ax}=3.75m		LHD R_{ax}=3.6m	
	Calc.	inter-polation	Calc.	inter-polation	Calc.	inter-polation
BD	0.9601	←	1.0184	←	1.0791	←
$\langle a_p \rangle$	0.535	0.552	0.589	0.593	0.636	0.584
ι_0	0.432	0.402	0.349	0.334	0.378	0.306
ι_a	0.964	1.049	1.214	1.257	1.571	1.229
R_{ax}	3.9	3.903	3.75	3.767	3.6	3.651
R_{geo}	3.816	3.811	3.740	3.738	3.672	3.694