Low temperature and high-density ignited operation in FFHR helical reactor

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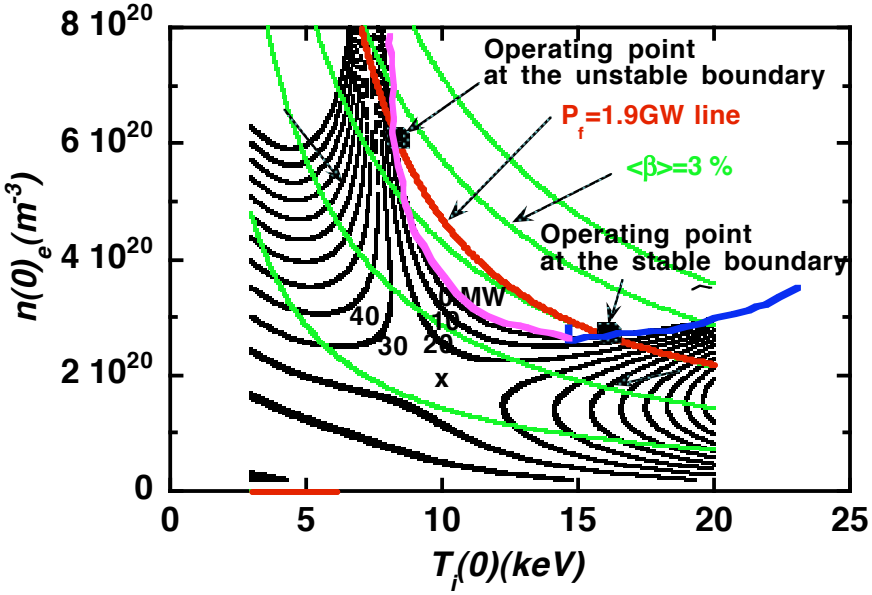
In collaboration with Profs. A.Sagara, R.Sakamoto, and O.Motojima et al.

1. Motivation of this study:

Recently, super dense core plasma with $n(0)\sim1\times10^{21}$ m^{-3} has been achieved in LHD device with pellet injection experiments. So low temperature and high-density operation is desirable in FFHR helical reactor if it is possible.

In general an operation in this high-density regime suffers thermal instability. We have discovered how to control this instability.



Stabilization of the unstable point is possible by feedback control of, for example, a reverse pendulum.

Our previous study concentrated on the stable operation point.

Many stabilizing methods of the thermally unstable ignition point have been proposed so far, by [1] fueling, [2] impurity injection, [3] heating power modulations, and [4] these combinations.

In these control algorithms, 0-dimensional equations of the particle and power balance equations have been linearized around the unstable operating point, and then stabilizing technique have been applied by

[1] H-infinity control,
[2] non-linear control method, and
[3] neural network control.

Disadvantage:

[1] Linearization is necessary in many equations and hence it may be difficult to apply it to the actual situation.
[2] Only stabilization around the unstable operating point has been shown. The access to the unstable operating point from the zero temperature and density has not been demonstrated.

We have discovered a new PID control algorithm to stabilize a thermally unstable operation for FFHR.

2. 0-D equation and control algorithm

2.1.Profiles

[1] Thermally stable operation to unstable operation

@Parabolic density profile: α_n=1.0 : $n(x)/n(0) = (1-x^2)^{\alpha_n}$

@Parabolic temperature profile: α_T=1.0 : $T(x)/T(0) = (1-x^2)^{\alpha_T}$

[2] SDC type operation $\mathrm{Tanh}(x)=(e^x-e^{-x})/(e^x+e^{-x})=1-2e^{-x}/(e^x+e^{-x})$

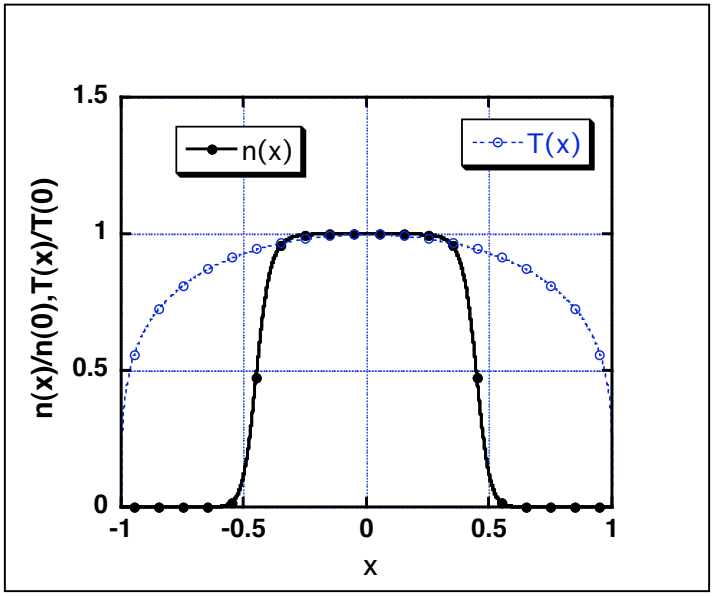
@Box type density profile: $n(x)/n(0) = 0.5[1-\mathrm{Tanh}(20(x^2 - a_n))]$

$[\mathrm{row}/a]_{ITBFM}$=0.45 (a_n=0.2)

@Parabolic temperature profile: α_T=0.25 : $T(x)/T(0) = (1-x^2)^{\alpha_T}$

This profile is necessary to obtain
@1.9 GW fusion power
@$n(0)\sim1\times10^{21}$ m^{-3}

in R=14 m FFHR.



2.2. Power balance calculation:

Box type density profile : $n(\rho) = n(0) \times 0.5\left[1 - Tanh\left\{20\left(\rho^2 - 0.2\right)\right\}\right]$

Parabolic temperature profile: $T(\rho) = T(0)\left(1 - \rho^2\right)^{0.25}$

@Volume averaged plasma energy:

$$\bar{W} / 3kn(0)T(0) = \int_0^1 0.5\left[1 - Tanh\left\{20\left(\rho^2 - 0.2\right)\right\}\right] \times \left(1 - \rho^2\right)^{0.25} 2\rho d\rho$$

$$= \frac{1}{1 + \alpha_n + \alpha_T} \Rightarrow 0.194429$$

@Volume averaged Bremsstrahlung loss:

$$\bar{P}_B / \left\{1.488 \times 10^{-38} Z_{eff} n(0)^2 \sqrt{T(0)}\right\} = \int_0^1 \left\{0.5\left[1 - Tanh\left\{20\left(\rho^2 - 0.2\right)\right\}\right]\right\}^2 \times \left(1 - \rho^2\right)^{0.25/2} 2\rho d\rho$$

$$= \frac{1}{1 + 2\alpha_n + 0.5\alpha_T} \Rightarrow 0.172891$$

@Volume averaged alpha heating power :

--> direct integration

@Plasma conduction loss: ISS95 scaling

$$\tau_E[s] = \gamma_{HH} \times 0.079 \iota_{2/3}^{0.4} \bar{n}_{19}^{\;0.51} [\times 10^{19} m^{-3}] B_o^{0.83} [T]$$

$$\times \bar{a}^{2.21} R^{0.65} [m] / \left\{ \left(P_{NET} - dW/dt \right) 2\pi R \pi \bar{a}^2 \times 10^{-6} \right\}^S [MW] = \frac{C_{op} / \left(2\pi R \pi \bar{a}^2 \right)^S}{\left(P_L \times 10^{-6} \right)^S} = \frac{C_{op}'}{\left(P_L \times 10^{-6} \right)^S}$$

Plasma conduction loss is obtained

$$P_L = \frac{\langle 1.5kTn \rangle}{\tau_E} = \frac{A_L n(0)}{\tau_E} = \frac{A_L n(0)}{C_{op}'} \left(P_L \times 10^{-6} \right)^S$$

where $A_L = \left[1.5e \left\{ f_D + f_T + 1/(T_i/T_e) + f_{alpha} \right\} T_i(0) / \left(1 + \alpha_n + \alpha_T \right) \right]$

The final plasma conduction loss and the confinement time are

$$@\ P_L = \left[\frac{A_L n(0)}{C_{op}'} \right]^{\frac{1}{1-S}} \left(10^{-6} \right)^{\frac{S}{1-S}} \propto \frac{T(0) n(0)^{1.196}}{\gamma_{HH}^{\;2.439}}$$

$$@\ \tau_E = \frac{A_L n(0)}{P_L} = \frac{\left(C_{op}' \right)^{\frac{1}{1-S}}}{\left[A_L n(0) \times 10^{-6} \right]^{\frac{S}{1-S}}} \propto \frac{\gamma_{HH}^{\;2.439}}{n(0)^{0.196} T(0)^{1.439}}$$

2.3.Control algorithm on heating power:

[1] Stable operating point

Feedback control of the heating power was done using the density limit scaling

$$\gamma_{DLM} n(0) \le n(0)_{\lim}[m^{-3}] = \gamma_{SUDO} \frac{0.25\times10^{20}}{\gamma_{pr}} \sqrt{\frac{\left\{P_{NET}[W]\times10^{-6}\right\} B_o[T]}{\bar{a}^2 R[m]}}$$

as

$$P_{EXT}[W] = \left\{\gamma_{pr} \frac{\left[\gamma_{DLM} n(0)[m^{-3}]\right]}{\gamma_{SUDO} 0.25\times10^{20}}\right\}^2 \frac{\bar{a}^2 R[m]}{B_o[T]} \times 10^6 - \left(P_\alpha - P_B - P_S\right)$$

This cannot be used in the unstable operation regime.

(For a large heating power, the density limit is increased, but the operating point enters into the sub-ignition regime due to heating power.)

[2] Unstable operating point

Pre-programming has been used at present

2.4. Control algorithm on continuous fueling:

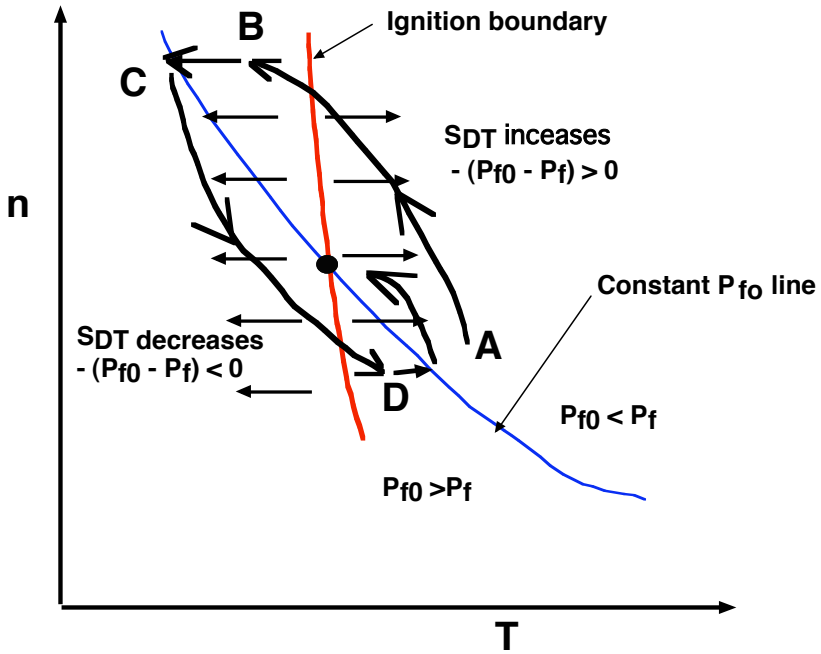
$$S_{DT}(t) = S_{DT0}\left\{e_{DT}(P_f) + \frac{1}{T_{\text{int}}}\int_0^t e_{DT}(P_f)dt + T_d\frac{de_{DT}(P_f)}{dt}\right\}G_{fo}(t) \qquad (1)$$

T_{int} the integration time, T_d the derivative time, **[Note: $S_{DT}(t)=0$ if $S_{DT}(t)<0$]**

The error the fusion power : $e_{DT}(P_f) = c(1- P_f/P_{fo})$

[1]c=+1 for the stable boundary,

[2]c= -1 for the unstable boundary,



@Stabilization mechanism

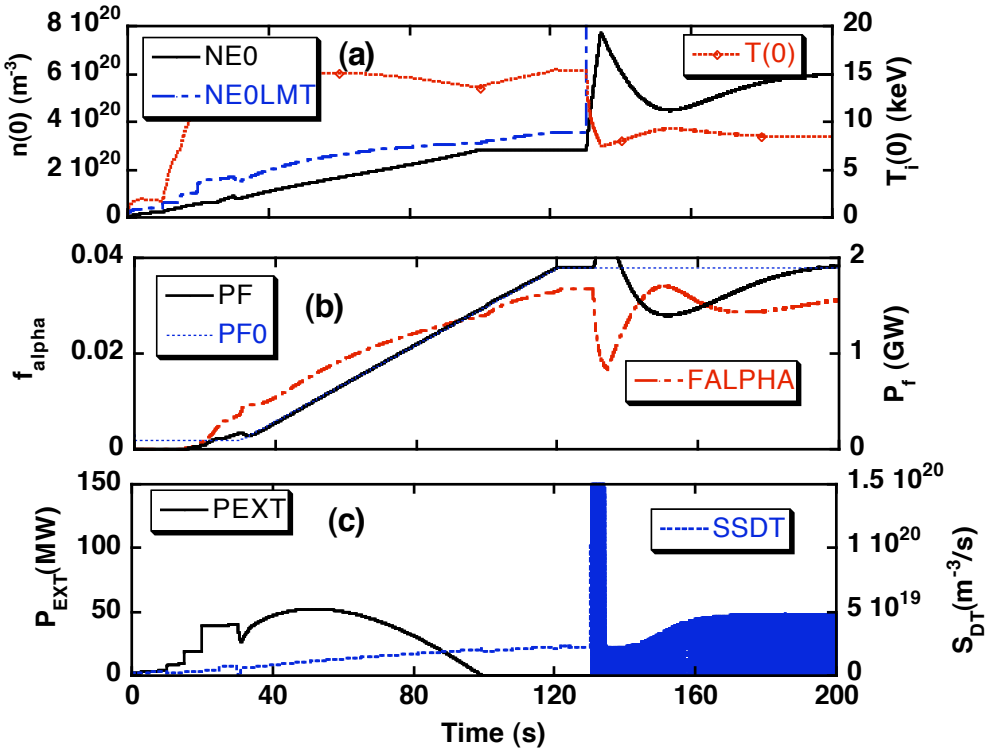
1. **Cooling by fueling**
2. **Operation path movement in the sub-and ignition regime**

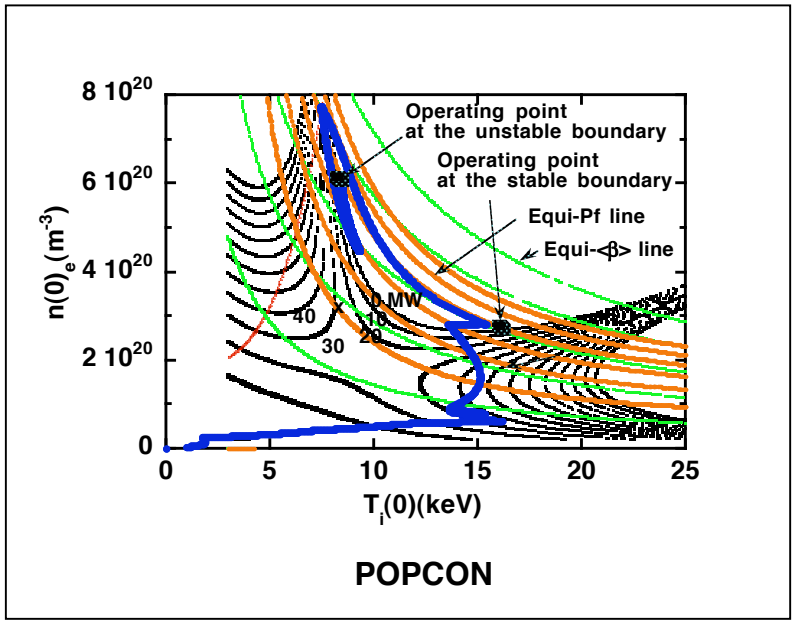
@ During ignition access, a stable control is possible.

3. Thermally stable to unstable operation regime with parabolic profiles

3.1. Late transition to the unstable regime.

R=14m, a_{eff}=1.72m, B_o=6.0 T, η_α=0.90, γ_{HH}(ISS95)=1.92 (γ_{LHD}=1.2), P_f=1.9 GW
α_n=1.0, α_T=1.0, τ_α^*/τ_E=3, τ_p^*/τ_E=3, γ_{SUDO}=4.5~7.5

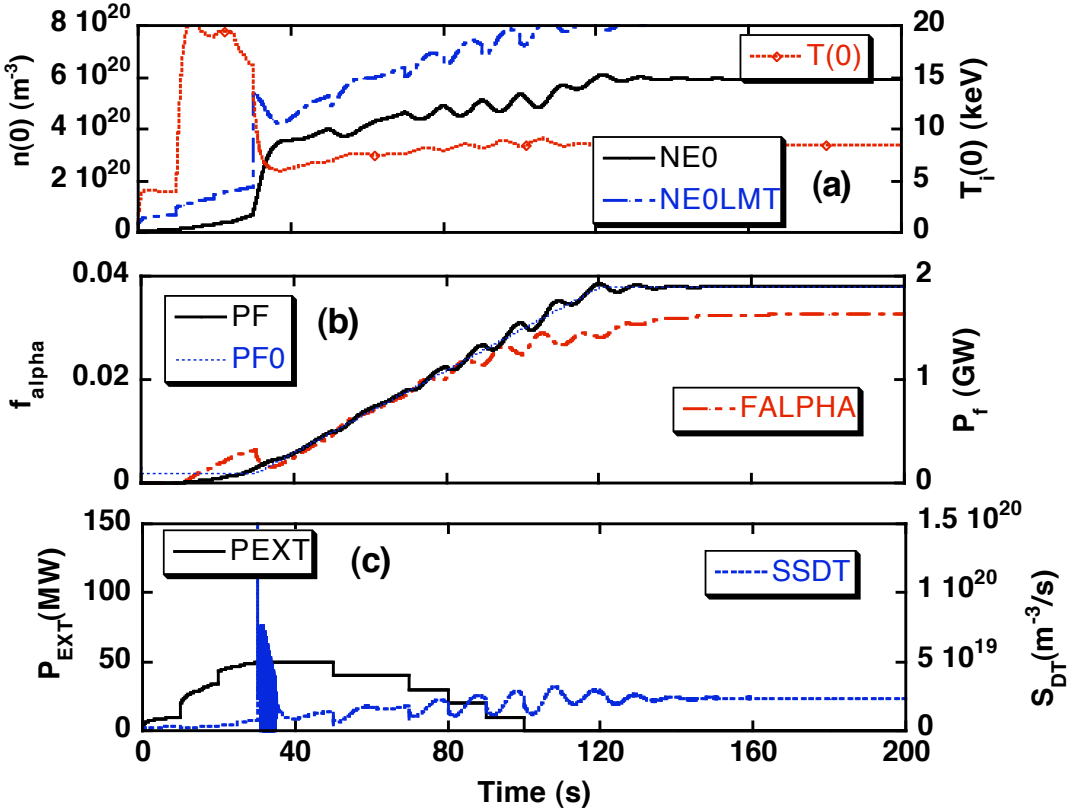


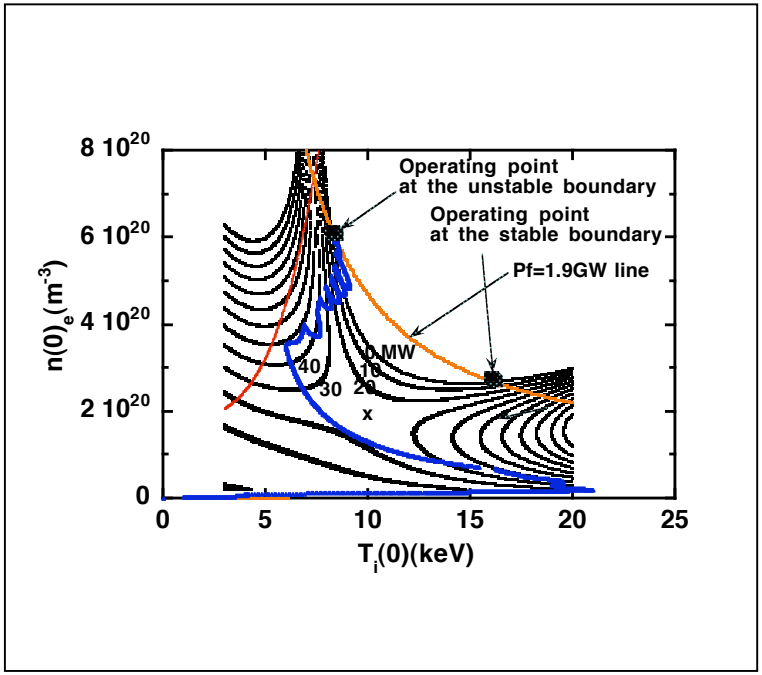


After reaching the stable point, the change in c=+1 to -1,and setting of $P_{f0}<P_f$ ($e_{DT}(P_f)=-c(1-P_f/P_{fo})>0$) can supply fuel and move the operating point to the unstable ignition boundary. (γ_{SUDO} =7.5 in this case)

3.2. Early transition at 30 sec to the unstable regime.

T_d=1s, T_{int}=10s, γ_{SUDO} =4.5

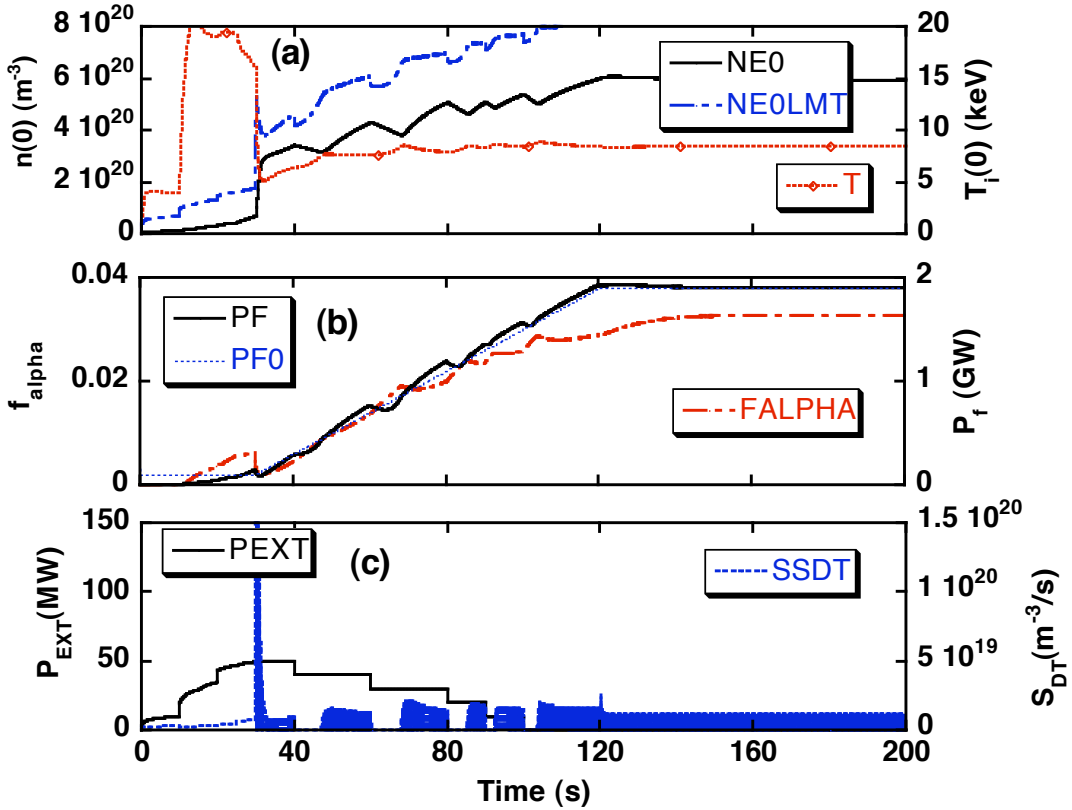




3.3. Long particle confinement time:

Density control is important in unstable operation. Even if particle confinement time is long, it is possible to control because the effective particle confinement time is shorter due to fusion reactions

$\tau_\alpha^*/\tau_E=3$, $\tau_p^*/\tau_E=15$, $T_d=5s$, $T_{int}=10s$:



For combined electron density equation, particle confinement time is determined by τ_p^* and τ_α^*.

$$\frac{dn_e(0)}{dt}=\frac{1}{1-8f_o}\left[(1+\alpha_n)S_{DT}(t)-\left\{\frac{f_D+f_T}{\tau_p^*}+\frac{2f_\alpha}{\tau_\alpha^*}\right\}n_e(0)\right]$$

In general, for D-T separate equation

$$\frac{dn_D(0)}{dt}=(1+\alpha_n)S_T(t)-\left\{\frac{(1+\alpha_n)n_T(0)\overline{<\sigma V>_{DT}(x)}\tau_D^*}{\tau_D^*}+\frac{1}{\tau_D^*}\right\}n_D(0)$$

$$=(1+\alpha_n)S_T(t)-\frac{n_D(0)}{\tau_{Deff}^*}$$

$$\tau_{Deff}^*=\frac{\tau_D^*}{(1+\alpha_n)n_T(0)\overline{<\sigma V>_{DT}(x)}\tau_D^*+1}$$

[1] $\tau_p^*/\tau_E=3$:

$\tau_E=4.7s$, $\tau_p^*=14$ s, $\tau_{Deff}^*=12.3$ s

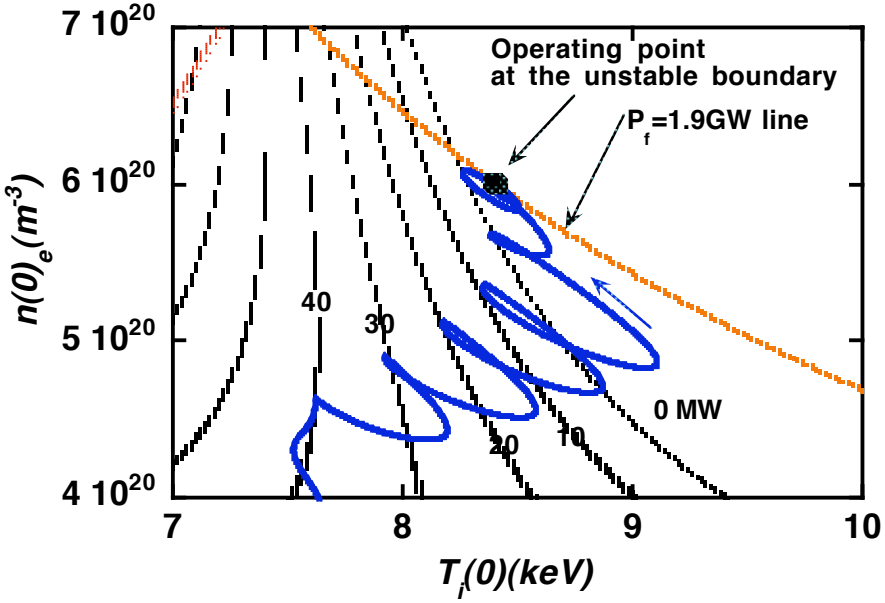
[2] $\tau_p^*/\tau_E=8$:

$\tau_E=4.7$ s, $\tau_p^*=37$ s, $\tau_{Deff}^*=27.1$ s

3.4.Characteristics of the operating point at the thermally stable and unstable ignition boundary.

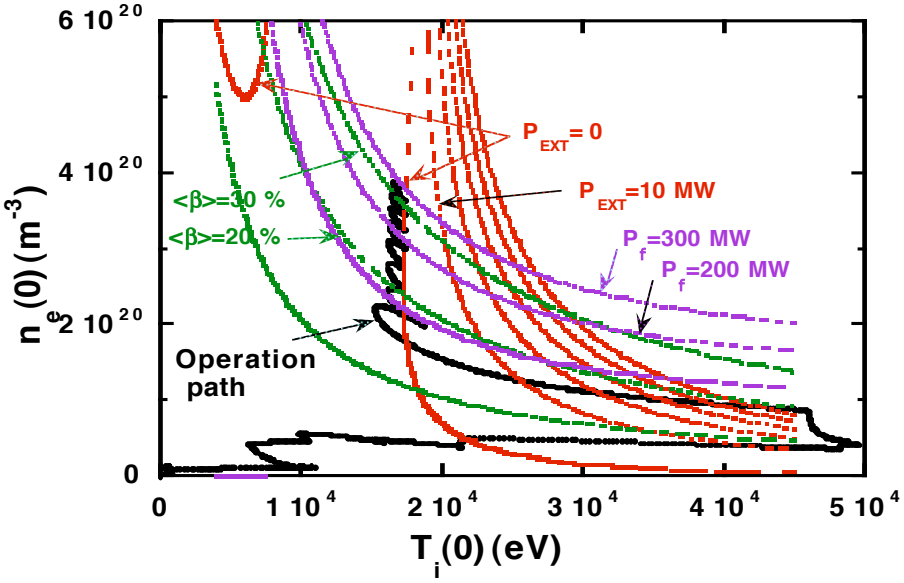
[1] Around the unstable boundary
FFHR (ISS95)

Direction of the operating point: CCW --> c=-1



[2] Around the stable boundary
CTF (IPB98y2)

CW--> c=+1



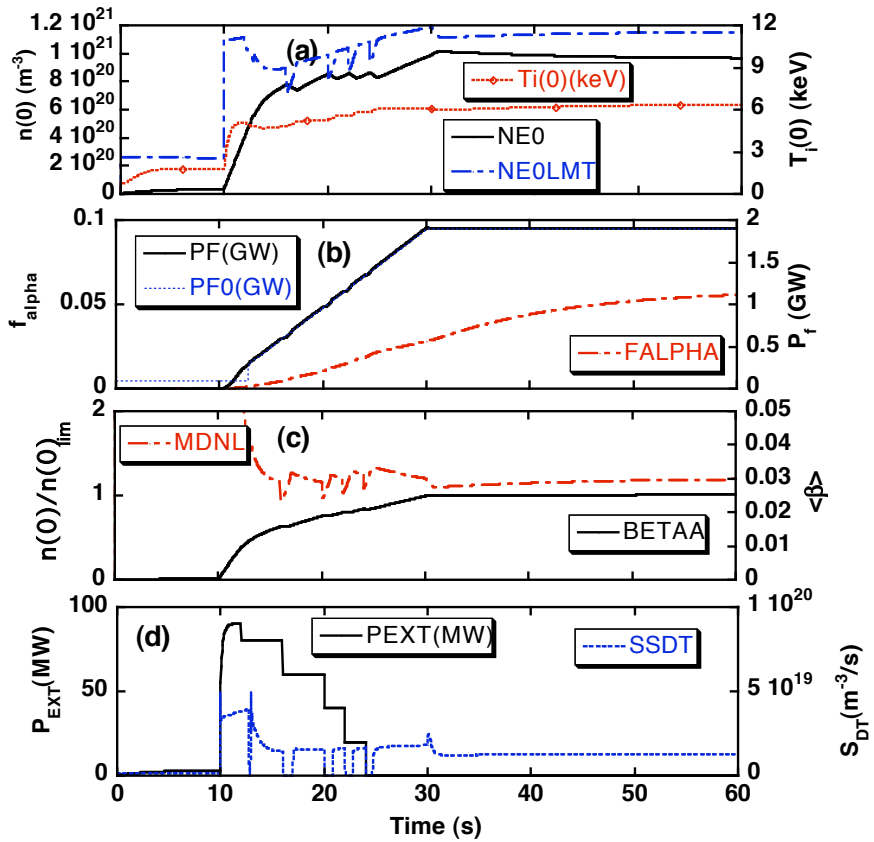
Comparison with stable and unstable operating point

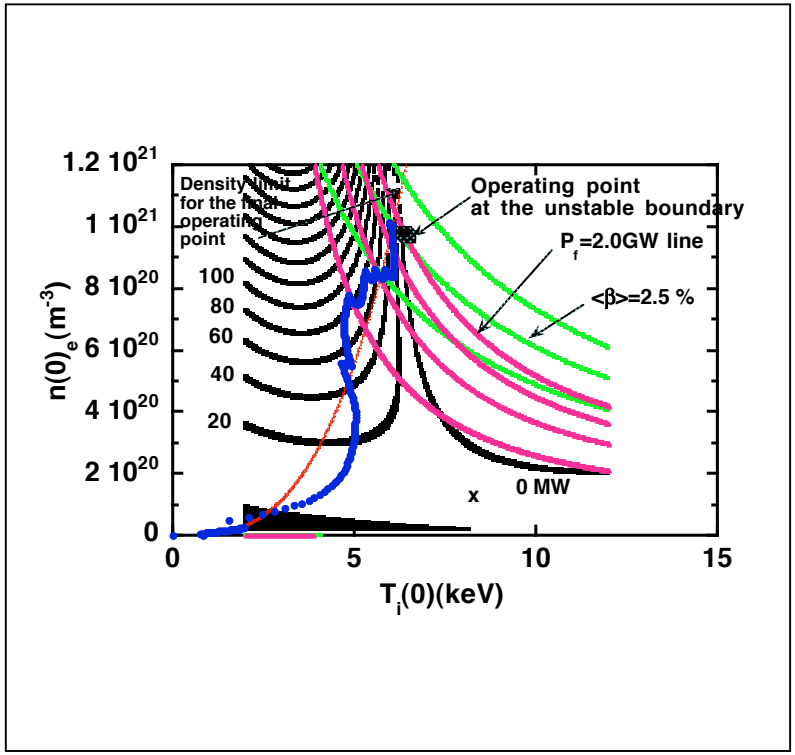
		Steady state value	
		Stable operating point	Unstable operating point
Major radius	R (m)	14.0	
Effective minor radius	$\bar{a}$ (m)	1.73	
Polarity/Field period	ℓ /m	2/10	
Coil pitch parameter	g	1.15	
Magnetic filed	B_o (T)	6.0	
Maximum magnetic field	B_{omax} (T)	13.3	
Coil magnetic energy	W_c(GJ)	120	
Blanket thickness	ΔB(m)	1.2	
Rotational transform	$\iota_{2/3}$	0.92	
Maximum external heating power	P_{EXT} (MW)	100	
Confinement factor over ISS95 scaling	γ_{ISS95}	1.92	
Confinement time	τ_E (s)	1.9	3.9 (4.70 at SDC)
Helium ash fraction	F_{alpha}	0.034	0.034 (0.058)
Effective ion charge	Z_{eff}	1.48	(1.52)
Operation density	$n_e(0)$ (10^{20} m^{-3})	2.8	6.0 (9.6)
Density limit factor over Sudo scaling	γ_{SUDO}	1.5	4.5 (5.5)
Density limit margin in the steady state	$[n(0)_{limit}/n(0)]$	1.27	1.36 (1.59)
Ion temperature	$T_i(0)$ (keV)	15.3	8.5 (6.4)
Beta value	$<\beta>$(%)	3.0	3.6 (2.5)
Fusion power	P_f (MW)	1900	
Neutron power	P_n (MW)	1520	
Alpha heating power	P_α (MW)	380x0.9	
Bremsstrahlung power	P_B (MW)	57	181 (248 : P_B/P_a= 72%)
Synchrotron radiation power	P_S (MW)	3.4	0.97 (0)
Plasma conduction power	P_L (MW)	282	160 (96)
Electric power output (thermal efficiency)	P_e (MW)	627(33%)	
Neutron wall loading	Γ_n (MW/m^2)	1.5	
Heat flux to first wall	Γ_h (MW/m^2)	0.06	0.18 (0.25)
Heat flux to divertor for 0.1m wet plate width	Γ_{div} (MW/m^2)	16	9.1 (5.4)

4. Low temperature and high-density operation with SDC profile

Continuous fuelling: γ_{HH}(ISS95)=1.6 (γ_{LHD}=1.0), τ_α^*/τ_E=3, τ_p^*/τ_E=3, γ_{SUDO}=5.5,

T_d=1s, T_{int}=10s, P_{EXT}=80MW, n(0)=9.62x10^{20} m^{-3}, T_i(0)=6.4 keV, f_α=5.84 %, <β>=2.5 %(at 200s)





10<t<12.8 sec : $e_{DT}(n)=+(1-n/n_o)$ **Density control**

12.8 sec <t : $e_{DT}(P_f)= -(1-P_f/P_{fo})$ **Fusion power control**

5. Pellet injection simulations with SDC profile

$$Error(P_f) = \left\{ e_{DT}(P_f) + \frac{1}{T_{\text{int}}} \int_0^t e_{DT}(P_f)dt + T_d \frac{de_{DT}(P_f)}{dt} \right\}$$

$$\begin{cases} S_{DT}(t) = S_{DTpellet} & for\ \ Error(P_f) > 0 \\ S_{DT}(t) = 0 & for\ \ Error(P_f) \le 0 \end{cases}$$

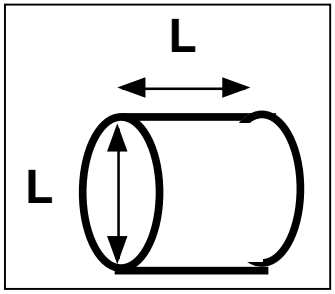
@Plasma Volume V_p=827 m^3

@DT solid molar volumes = 19.88 mm^3/mol [P.C.Souers, "Hydrogen Properties for Fusion Energy"p80, University of California Press,]

@DT ice particle number density={6.02x10^{23}x2}/19.88 [mm^3/mol]=6.05x10^{28}/m^3

[1]L=12mm $N_{pell} = \left\{ \pi \left(L_p / 2 \right)^2 L_p \right\} \times 6.05 \times 10^{28} = 82 \times 10^{21}$

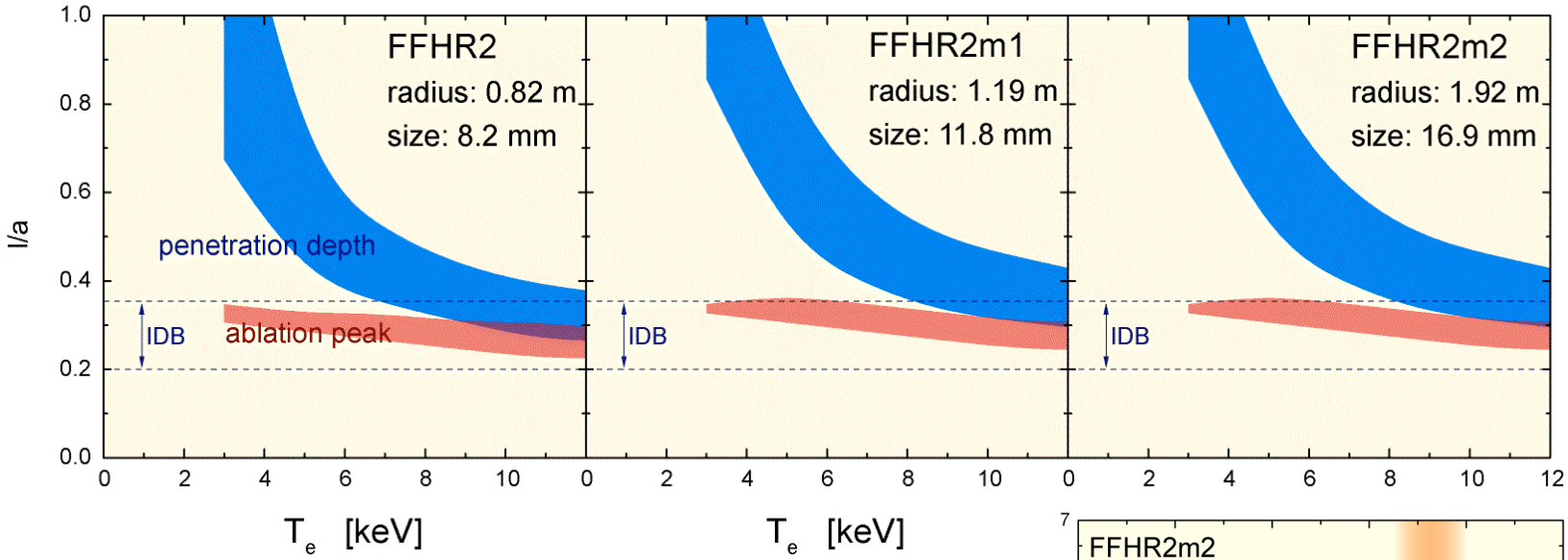
->S_{DTpell}=N_{pell}/V_p=9.92x10^1 9/m^3

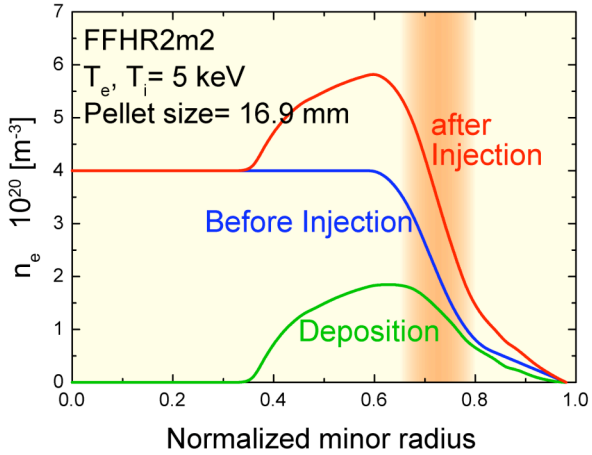


[2]L=16mm :

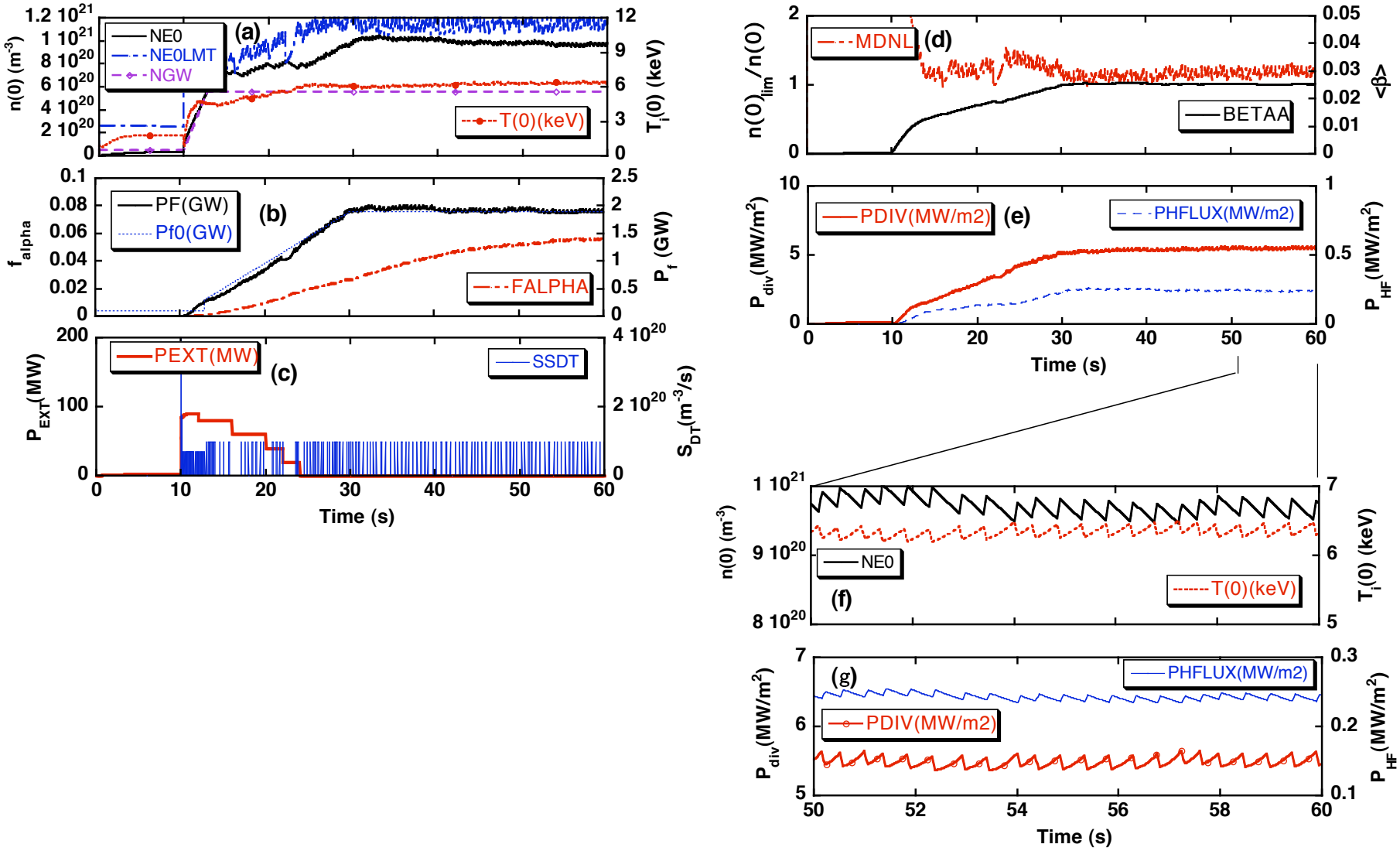
N_{pell}=194x10^{21} /m^3 −＞ S_{DTpell}=N_{pell}/V_p=23.53x10^{19}/m^3

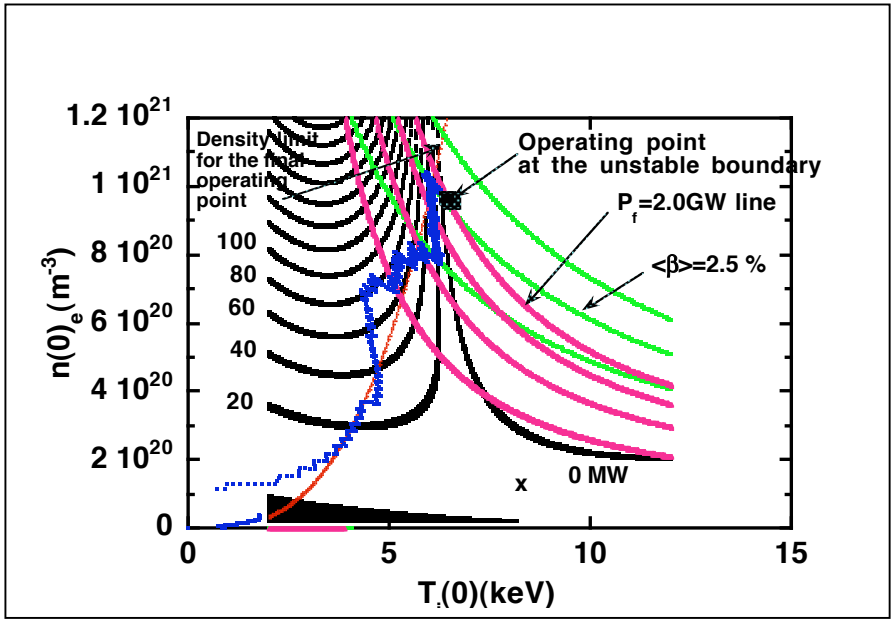
Sakamoto 1-D calculation





[1] L=12 mm pellet injection (T_d=0.26, T_{int}=8s)





@Controllable for density variation of $\Delta n \sim 2.5 \times 10^{19} m^{-3}$

@Divertor heat flux variation

$\Delta P_{div}/P_{div} \sim 0.1/5.5 = 3.6\%$

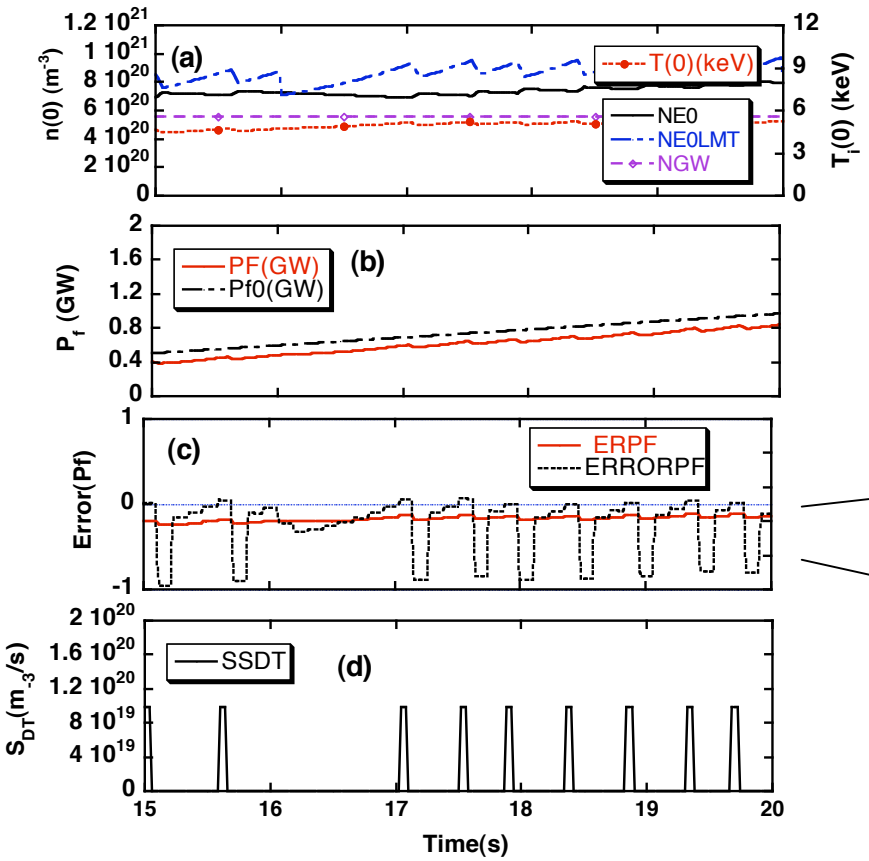
@First wall heat flux variation

$\Delta P_{HF}/P_{HF} \sim 0.01/0.24 = 4.1\%$

Fuel control during the fusion power rise-up phase：

Compared to continuous fueling, careful adjustment of T_d is necessary

（T_d=0.26s,T_{int}=8s）



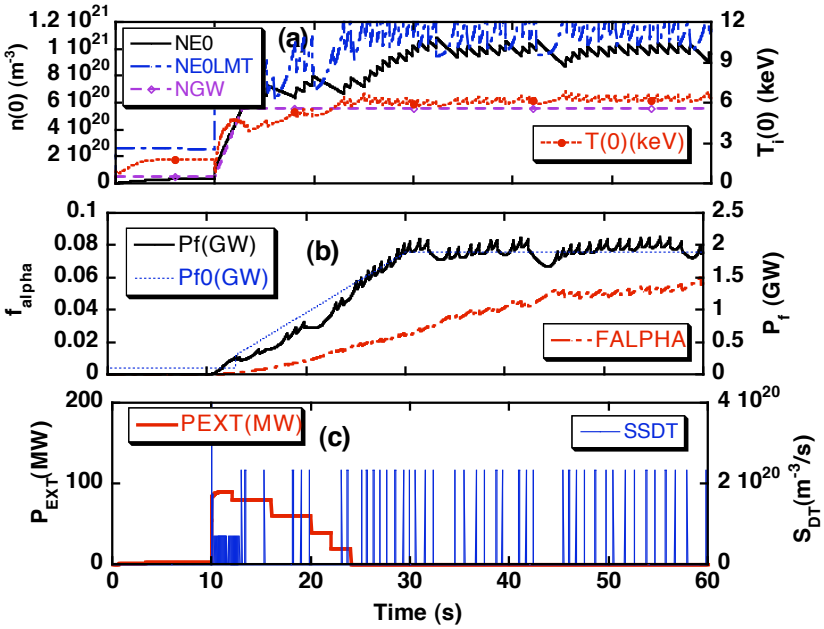
For a long time during the fusion power rise-up phase, P_f<P_{f0} holds and no fueling by P-control alone.

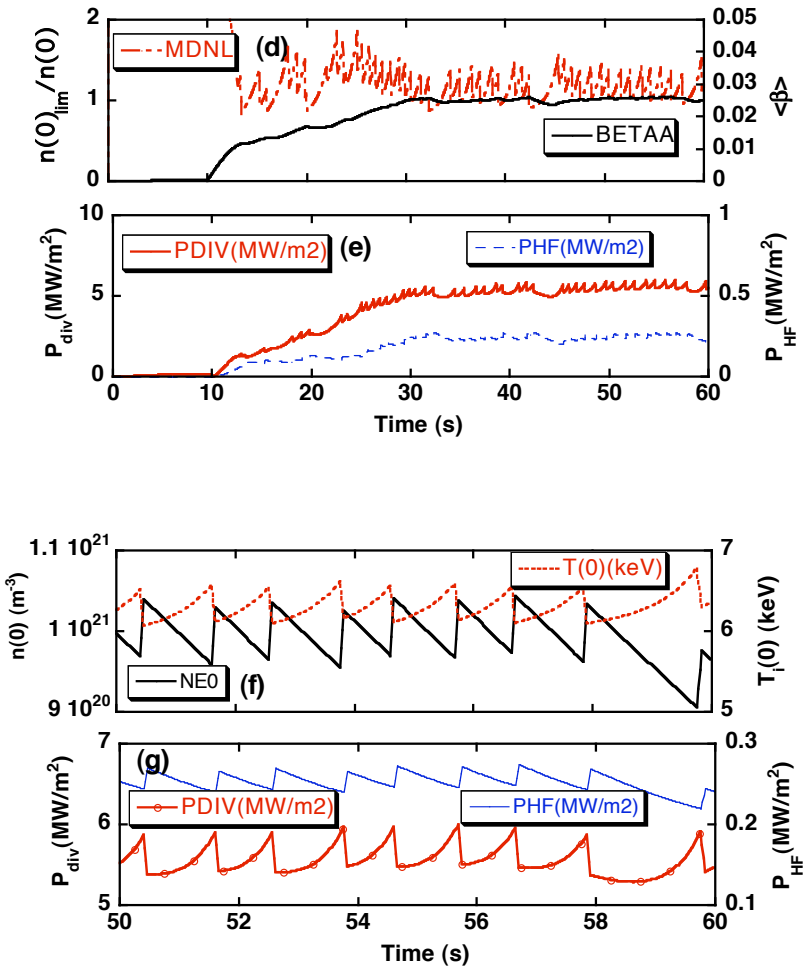
--> Derivative time T_d is important

$$ERPF = \left\{ e_{DT}(P_f) \right\}$$

$$ERRORPF = \left\{ e_{DT}(P_f) + \frac{1}{T_{\text{int}}} \int_0^t e_{DT}(P_f)dt + T_d \frac{de_{DT}(P_f)}{dt} \right\}$$

[2] 16 mm pellet injection (T_d=0.26, T_{int}=8s)





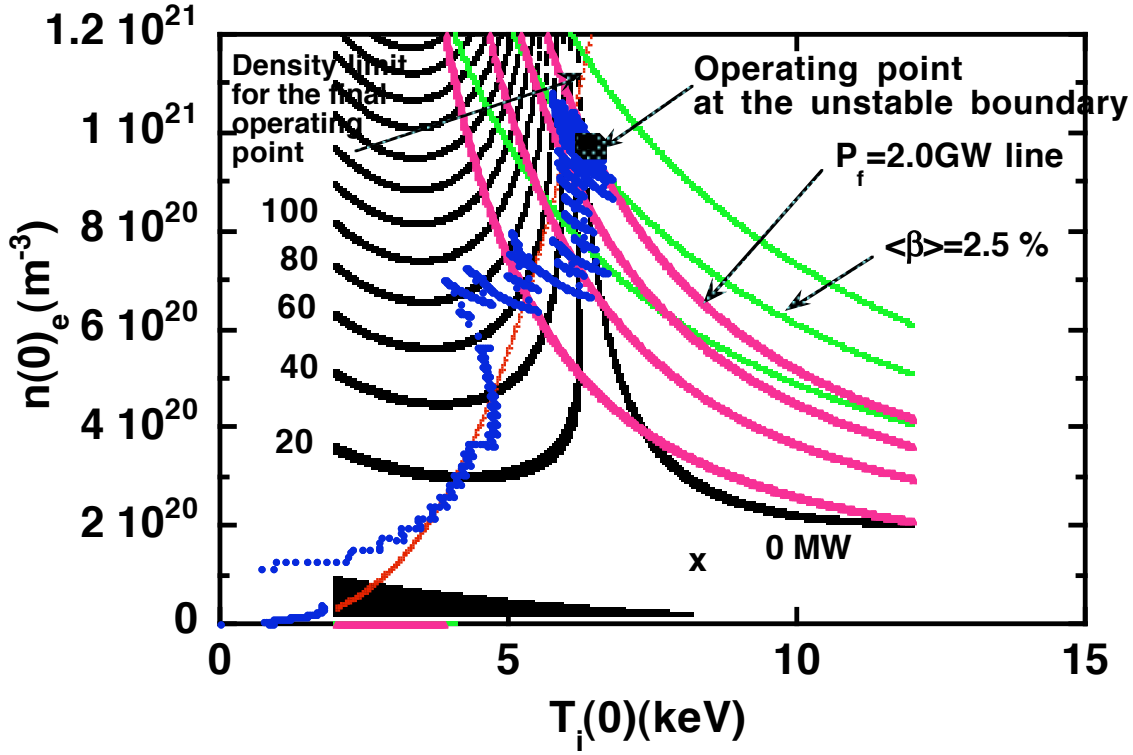
@Controllable even for density variation of $\Delta n \sim 8\times10^{19} m^{-3}$

@Divertor heat flux variation

$\Delta P_{div}/P_{div} \sim 0.5/5.5 = 9\%$

@First wall heat flux variation

$\Delta P_{HF}/P_{HF} \sim 0.025/0.24 = 10\%$

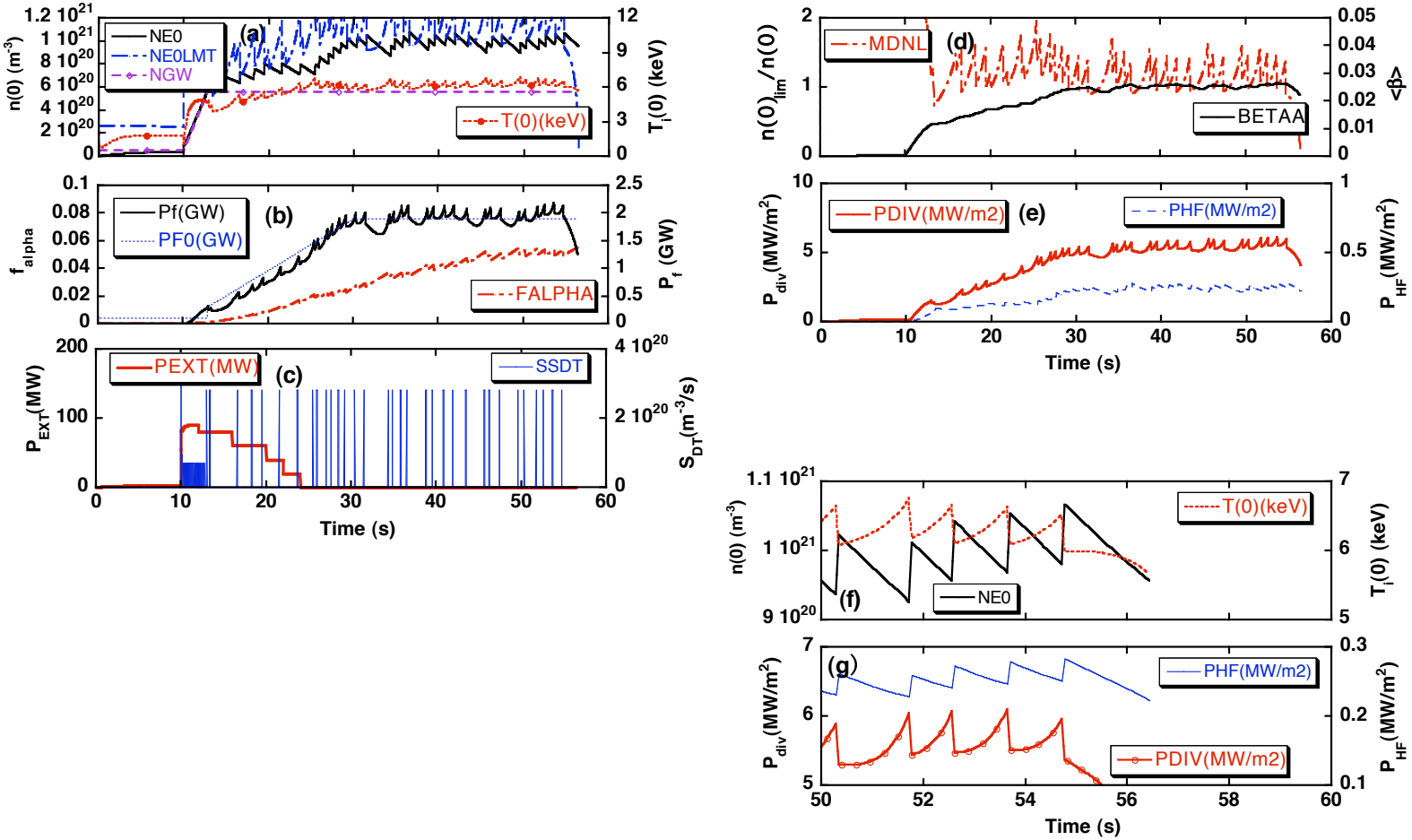


Operating point moves along the constant beta line (Constant W_p line).

Therefore, only fusion power can control this unstable ignition.

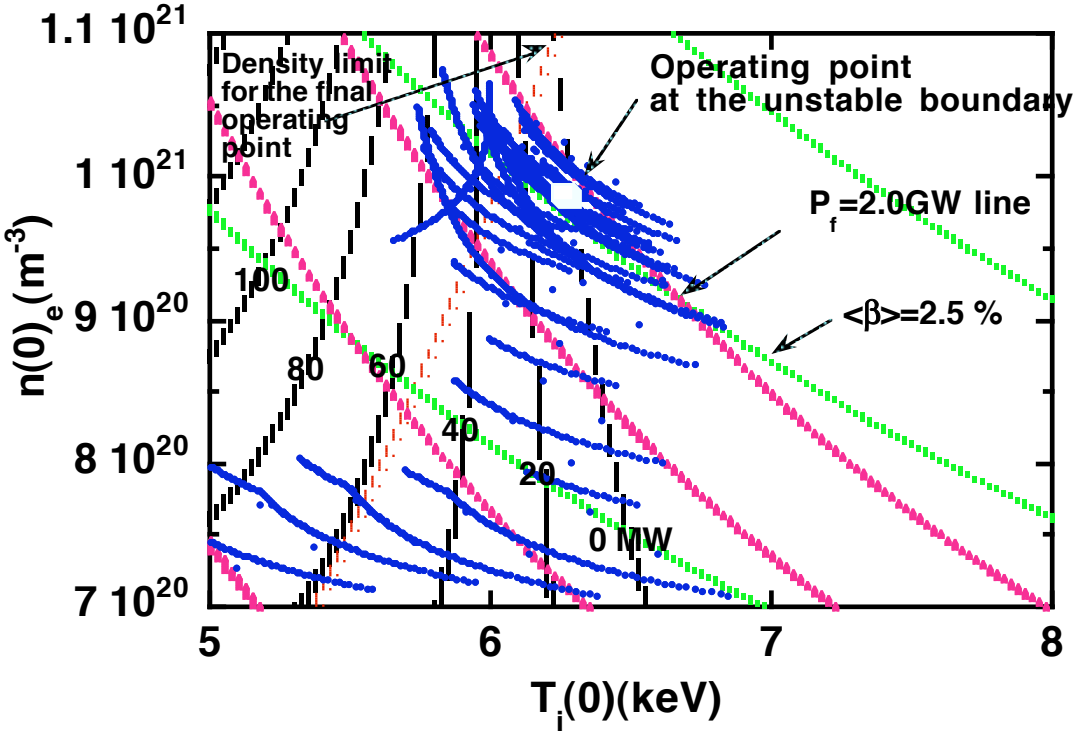
[3] 17 mm pellet injection (T_d=0.26, T_{int}=8s)

Ignition is terminated in the steady state.

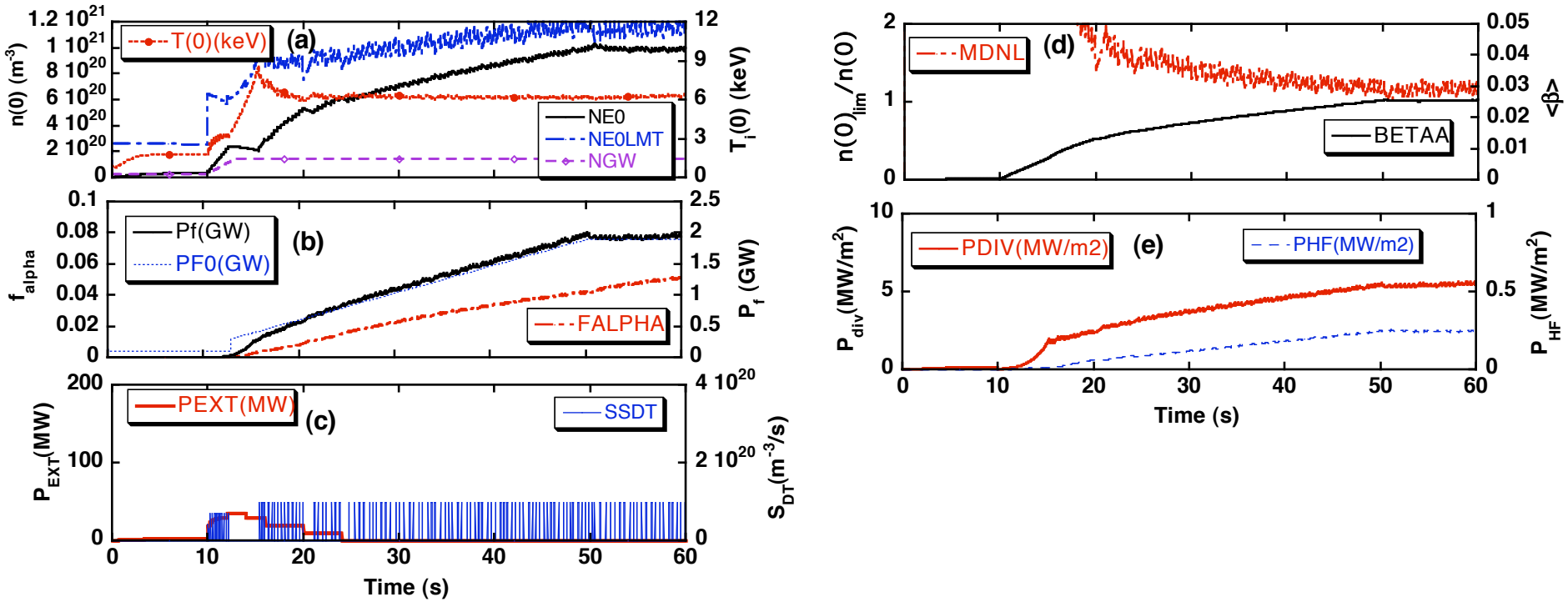


@Uncontrollable after too much density variation $\Delta n > 8 \times 10^{19} m^{-3}$

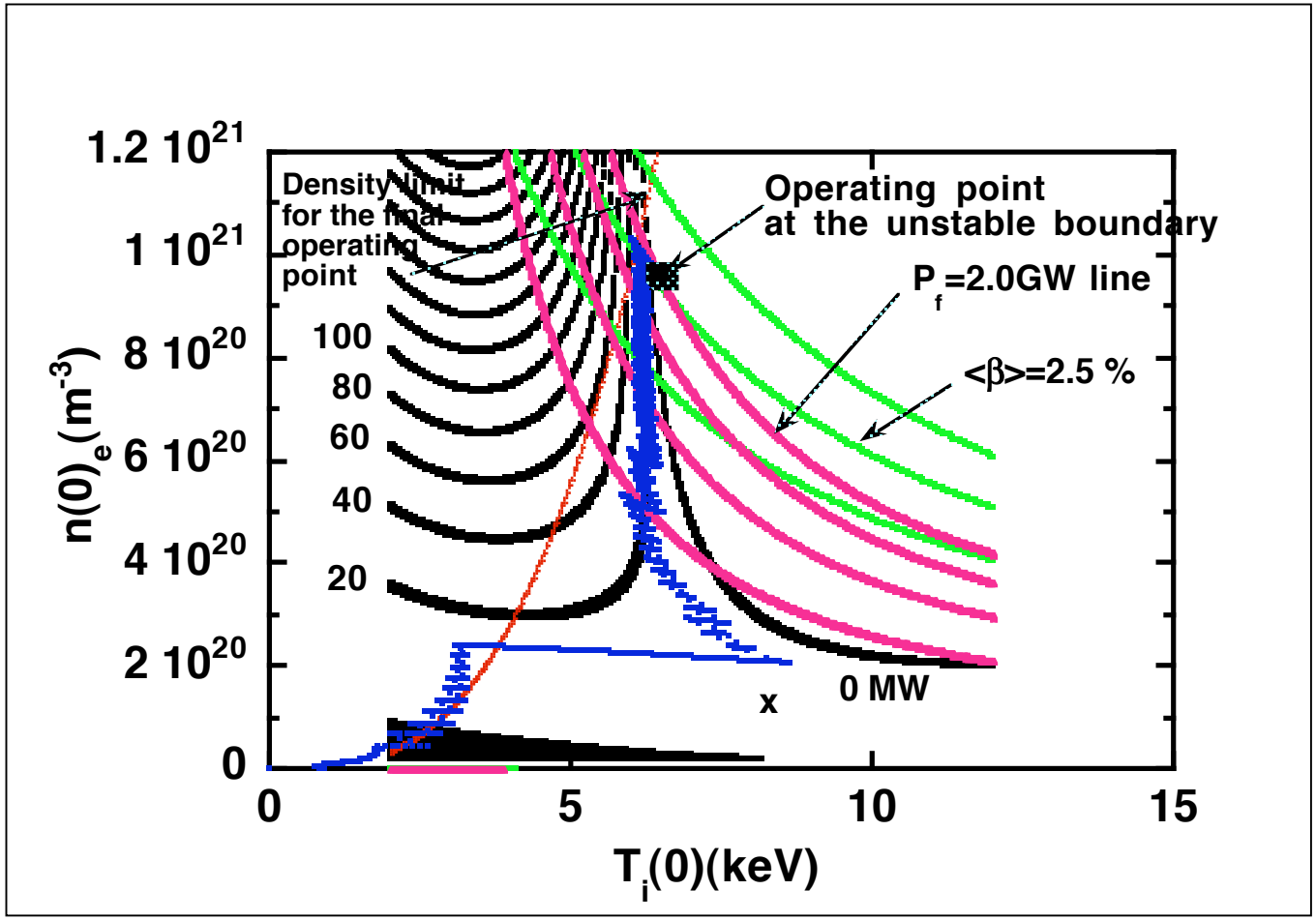
@When the density and temperature decrease at the same time, ignition is terminated.



6. Minimum heating power to ignition of 35 MW



When the initial density is decreased by density control, the heating power can be reduced to 35 MW

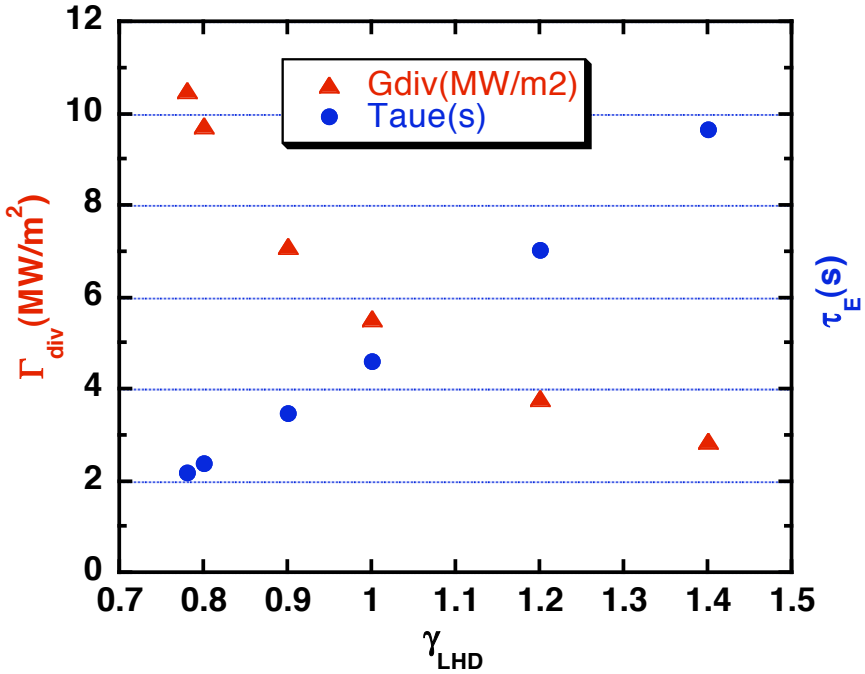

1.2 10^21
1 10^21
8 10^20
6 10^20
4 10^20
2 10^20
0
n(0)_e (m^-3)
0
5
10
15
T_i(0)(keV)
Density limit
for the final
operating
point
100
80
60
40
20
0 MW
x
Operating point
at the unstable boundary
P_f=2.0GW line
<β>=2.5 %

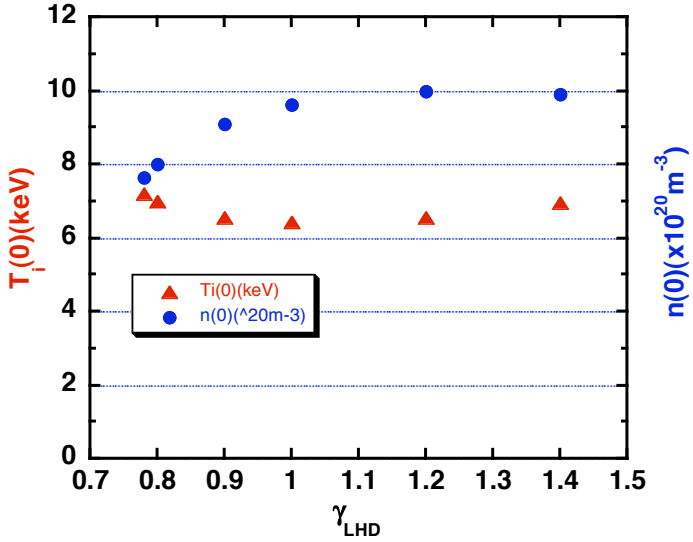
7. Further issues on reactor physics

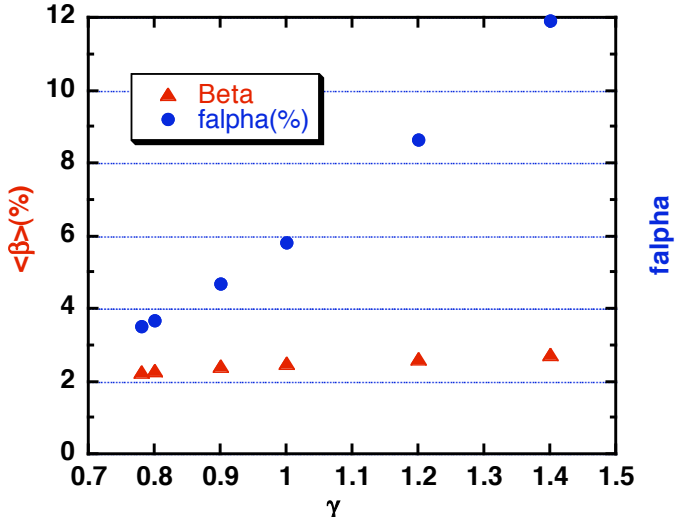
[1] ISS95 confinement scaling

@Ignition exists down to γ_{LHD}=0.8, or γ_{ISS95}=1.6x0.8=1.28.

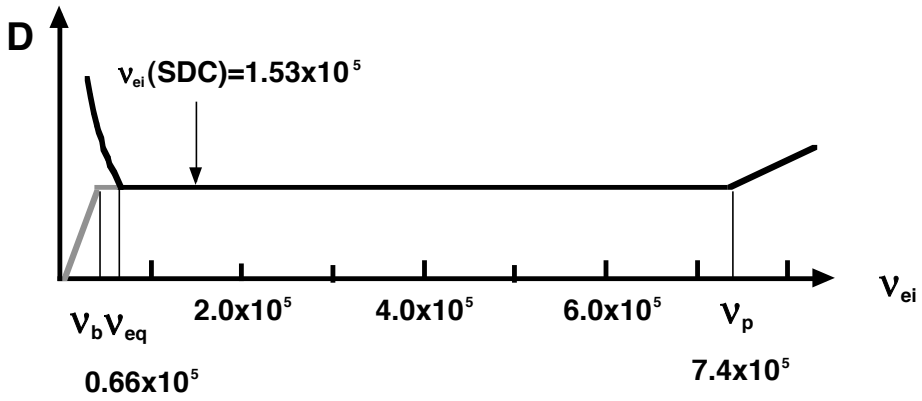
@Divertor heat flux~10 MW/m^2 at γ_{LHD}=0.8







[2] Plateau regime with respect to the collision frequency



@ No Neo-classical transport problems

@ Lower bootstrap current ?

$$v_{ei}[1/s] = 4.2\times10^{-12}\frac{n_e[m^{-3}]\ln\Lambda_{ei}}{T^{3/2}[eV]} = 1.53\times10^5\ [1/s],\quad v_p = 1/R(\iota/2\pi)V_e = 7.4\times10^5\ [1/s],$$

$$V = \sqrt{T_e/m_e} = 4.19\times10^5\sqrt{T_e(eV)} \qquad v_{eq} = \varepsilon_h^{3/2}v_p = 0.66\times10^5\ [1/s]$$

ε_h=0. 2

[3] Alfvén instability is expected (suggested by Dr Tobita): V_α~3.3V_{AV}

@Alfvén velocity of SDC plasma: n_i=n_e/2, A=(2+3)/2=1.5

$$V_{AV} = \frac{B}{\sqrt{\mu_o M_i n_i}} = 2.18\times10^6\frac{B}{\sqrt{An_i/10^{20}}} = 2.18\times10^6\frac{6}{\sqrt{1.5\times(1\times10^{21}/2)/10^{20}}} = 4.77\times10^6[m/s]$$

@Alpha particle velocity: $V_\alpha = \sqrt{\frac{3kT_\alpha}{M_\alpha}} = 1.695\times10^4\sqrt{\frac{T_\alpha(eV)}{A_\alpha}} = 1.695\times10^4\sqrt{\frac{3.5\times10^6}{4}} = 15.8\times10^6[m/s]$

8. Summary

[1] PID control of fueling based on the fusion power signal with sign of "-" can stabilize the unstable operation point .

[2] Access to the operating point from 0-density and temperature is possible. Parameter variation is allowed in some range.

[3] It is possible to control the unstable operating point during pellet injections. For R=14 m reactor, 12~16 mm size pellets are allowed for ignition. But the heat fluxs to divertor and first wall oscillate.

[4] In a high-density ignition regime, the confinement time is long, and then the plasma conduction loss is reduced. Therefore the divertor heat flux is reduced to 5.5 MW/m^2 for 10 cm plate width without impurity injection.

@Low temperature and high density operation is more advantageous than high temperature and low density operation in a tokamak.

[5] Although further issues on alpha, Bootstrap current, confinement scaling, remain, Engineering issues on assembling, maintenance etc may be most serious in present FFHR design.